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Blurred Boundaries

An Exercise in Wittgenstein's Method

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MCCOY: Oh, I know you, Mister Spock. You've never voiced it, but you've always thought that logic was the best basis on which to build command. Am I right?
SPOCK: I am a logical man, Doctor.
MCCOY: It'll take more than logic to get us out of this.
SPOCK: Perhaps, Doctor, but I know of no better way to begin. I realise command does have its fascinations, even under circumstances such as these. But I neither enjoy the idea of command, nor am I frightened of it. It simply exists. And I will do whatever logically needs to be done. Excuse me.

Star Trek - The Galileo Seven

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Introduction

> They used to make pickles, squashes, jams, curry powders and canned pineapples. And banana jam (illegally) after the FPO (Food Products Organization) banned it because according to their specifications it was neither jam nor jelly. Too thin for jelly and too thick for jam. An ambiguous, unclassifiable consistency, they said. (Roy, *The God of Small Things*, p. 30)

Jam and jelly can be found on any good breakfast table. They are ordinary things that are cooked, passed around, and stored in the fridge. In none of these interactions, the boundaries of the concept of *jam* or of *jelly* play any role. But it can come into view for special purposes. In this case, it is difficult to know what the lawmaker's intentions were. In the best case, its ultimate purpose is consumer protection and information, and an exact classification is needed for those purposes. How thick can a jam be before it becomes a jelly? For ordinary, practical purposes, no answer to this question is required. But the increased standards of exactness of the purposes of lawmaking may require engaging with it.

A difficulty that faces lawmakers is that the boundaries of ordinary concepts are blurred. The consistency of the fruit spread can be influenced by the amount of pectin, fruit, or water that is added, by the cooking duration and temperature, etc. Tampering with these factors slides the consistency of the fruit spread on a *gradual* scale; it can be thicker or thinner. But the characteristic of a predicative concept is to distinguish between those things that fall under it, and those things that do not; those things that are jams, and those things that are not jams. Predicative concepts are a yes-or-no affair. This does not mean that concepts are inapplicable to gradual scales; at the very least, one can impose an arbitrarily drawn sharp boundary. A pragmatic constraint for such a

stipulation is that it should be easily measurable, which is problematic in the case of the consistency of jams and jellies.

Nevertheless, the ordinary concept of *jam* does not neatly separate cases of jam from cases of not-jam by a sharp boundary. If no sharp boundary has been stipulated, it would be absurd to claim that the addition of a tiny crystal of pectin turns a jam into a jelly. Why should it be exactly that crystal, and not another? And there seems to be no way of finding out which one it is. However, if it is indeed the case that the ordinary concept of jelly is not sharply bounded, a paradox looms. The addition of a single crystal of pectin (and some stirring) does not turn a given pot of jam into a pot of jelly, we said. By this, we are not talking about a particular crystal; we generalise. The addition of *any* single crystal does not turn the jam into a jelly. But then, a whole series of adding single crystals of pectin will not turn the jam into jelly. But if so, we have in effect claimed that no amount of pectin turns jam into jelly, which is evidently false. This paradox can be succinctly stated with four statements and involves an argument which is valid in classical logic (see section 7.1). The main premise is the Tolerance principle, here formulated for the concept *jam*.

Tolerance: If a fruit spread that contains n crystals of pectin is a jam, then a fruit spread that contains $n + 1$ crystals of pectin is a jam as well.

This paradox was discussed by the ancient Greeks under the name of "sorites", and relates directly to the blurred boundaries of concepts. The absence of a sharp boundary implies that a small change in a relevant similarity dimension (amount of pectin) does not warrant a change in the applicability of a concept (jam). Clearly, a great number of small changes add up to a large change in that similarity dimension. And if the large change is seen as a totality of small changes, a change in the applicability of the concept seems to be unwarranted. But of course the large change should warrant a change in the applicability of the concept.

This phenomenon of blurred boundaries and the sorites paradox is discussed in contemporary philosophy under the title of "vagueness", and the concepts and statements concerned are called "vague". The technical use of these words is different than its standard one in English. In the standard sense, a speaker who makes a vague utterance is usually being unspecific. The statement "Measures have been taken to bring the economy of Greece on a growing path." could be criticised as being vague, on the basis that it is not specified *which* measures have been taken. Or vagueness can express unclarity

as in "There are vague memories in our souls of those misty centuries when the world was in its childhood." (Doyle, *A Study of Scarlet*, ch. 5). This unclarity is perhaps the strongest link to the philosophical sense. For vague concepts, not only is the boundary blurred, but there are also cases in the borderline region for which it is unclear whether they should be classified as F or as not-F. Regarding some fruit spreads in the transition zone between jams and jellies it is unclear whether they should be classified as jam or as jelly – their consistency is such that it would not be completely wrong to classify them as either. They are borderline cases of "jam" (or of "jelly"). (For more on unspecifity, see p. 62, for more on the technical use of vagueness, see section (3.1).)

Vagueness is by no means a new problem. In Roman and Greek antiquity, the sorites paradox was a matter of discussion between Stoics and Skeptics. The modern discussion can be said to have started in the twentieth century with Frege as a precursor in the late nineteenth century. However, it arose to prominence only after 1975 when the now classical articles of Dummett (1975) (a Frege scholar) and Fine (1975) were published. All three can be regarded as figures of Analytic Philosophy, the philosophical tradition that figured as the host of the subsequent voluminous discussion about vagueness. The story of why exactly this topic arose to such prominence cannot be told without considering the history of the relevant institutions, which I will not do. Nevertheless, there are three philosophical aspects of Analytic Philosophy that promote its interest in vagueness.

First, Philosophy, just as law-making, often aspires to standards of greater exactness. And this holds even more for Analytic Philosophy, which in its self-image strived to engage from a fresh perspective of clarity and exactness with the old philosophical problems, avoiding the errors of the past (cf. Glock 2008, pp. 168f). These general goals of clarity and exactness not only call for exact argumentation, but also for a clear presentation of the results. The latter seems to be at odds with vagueness, with the inability to locate a sharp boundary, with the inability to say of a case whether it is F or not. The goal then is to describe in an exact way the inexactness of vagueness.

Second, Analytic Philosophy has a special interest in language. In its beginnings, an important strand propagated the linguistic turn, which regarded the analysis of the structure of sentences as central for the resolution of philosophical problems and thus afforded the philosophy of language center stage (Wittgenstein's *Tractatus*, Carnap) (cf. Glock 2008, p. 121, 131). Carnap and Logical Positivism thereby represent the ideal language strand of the linguistic turn, which tried to systematise and improve on ordinary and technical languages in service of science. In contrast, Ordinary Language Philosophy

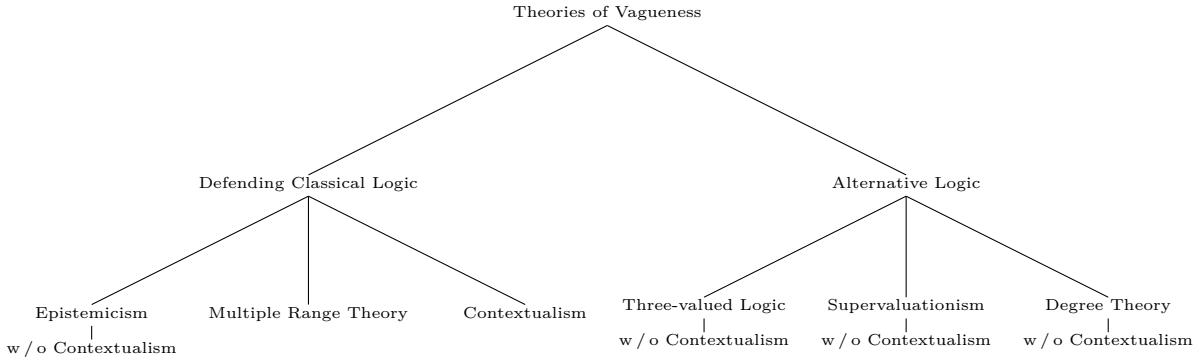
and the later Wittgenstein believe that to reform language is to change the topic and miss the initial problem or question. For Wittgenstein at least, the task of philosophy is to solve philosophical problems by engaging with the confusions and misunderstandings in the language in which they were brought into existence.

However, by the last third of the twentieth century, both Logical Positivism and Ordinary Language Philosophy had lost their popularity, and were replaced by a different kind of interest in language: the logification of ordinary language. From Ordinary Language Philosophy it takes interest in ordinary language. From Ideal Language Philosophy (with or without the linguistic turn) it takes the resources for constructing artificial, formal languages. The idea is that these formal methods afford a better understanding of ordinary language. An early outlier of that view is the *Tractatus* which claims that ordinary language fulfils the demands of a logically ideal language contrary to its appearance. But the basis for the dominance of that view was laid by Davidson in the sixties with the paper *Truth and Meaning*. He formulated a programme to replace an analysis of the concept of meaning by specifying the constraints and properties of a theory that would specify the meaning of every expression of a language (Davidson 2001, p. xv). While the first part of that programme, the elucidation of the concept of meaning, has receded into the background, the programme of constructing semantic theories along Davidsonian lines has become a new paradigm. This then, is the third aspect of Analytic Philosophy that explains its fascination with vagueness. How can vagueness be accommodated in a logicised view of ordinary language?

This viewpoint is reflected in how the main theories of vagueness view the problem and propose to solve it. The sorites paradox, which constitutes the heart of the matter, is seen as a threat for classical logic, which is the default option for a logicised view of ordinary language. The main theories can then be divided into those that defend classical logic by reinterpreting vagueness, and those that replace it by an alternative logic (see figure 1.

Williamson's epistemicism is the main defender of classical logic (1994). He conceives of vagueness not as a semantic phenomenon, but as an epistemic one. Vague concepts do have sharp boundaries, it is just that speakers do not know where the boundaries are. This of course is a special kind of ignorance, because it is radically unclear what further evidence could be drawn upon to locate the boundary between jams and jellies. For Williamson, the ignorance concerns the extension of concepts: There could be closely related concepts of *jam** and *jam*$^\circ$, whose extensions differ only in that the largest

Figure 1.: Theories of Vagueness



ratio of pectin for *jam** is 0.1224, and the largest pectin ratio of *jam*$^\circ$ is 0.1225. An important part of Williamson's theory is that the meaning of a word supervenes on and is determined by the use of that word. Then, since the speaker has no good overview over the use of a word in his linguistic community, he is in a position to know the basic meaning of his concept, but not its exact boundary. He does not know whether his word "jam" has the extension of *jam** or of *jam*$^\circ$.

This places the burden of the surprising sharpness on the relation between meaning and use. But it is highly questionable whether there is a working conception of that relation that supports epistemicism. There is an austere and a rich sense of "use". In the austere sense, only the applications (actual and counterfactual) of a concept count as use. In the rich sense, applications together with explanations, criticisms, and justifications count as use. But in the austere sense, it is doubtful whether use can determine (or constitute) meaning, where mistakes, irony, or jokes pose difficulties. Moreover, as Endicott (2000, ch. 6) argues, what should count as the relevant use of a linguistic community is itself vague (Who is a competent speaker? Which applications count? Do the applications have to be unanimous?). And a vague use can hardly determine a sharp meaning. In contrast, a richer conception of use which includes explanations entails that a speaker cannot be completely unaware of meanings of the words she uses. But ordinary speakers are evidently not aware of any sharp boundary of their concepts. Finally, it is at least possible that the use of a word consistently leaves certain cases indeterminate, such as Fine's (1975) nice numbers, where numbers below 13 are nice, and numbers larger than 17 are not nice, saying nothing about the numbers in between. Such an indeterminate use could not determine a determinate meaning. But then, an epistemicist has to argue that, as a matter of fact, use does not leave any cases indeterminate.

In the newest book-length proposal, Raffman (2014) also defends classical logic, though from a semantic perspective. According to her, the uncertainty regarding borderline cases stems from a systematic ambiguity of vague words. On her multiple-range theory, "jam" is ambiguous in that it expresses a number of closely related senses, each with a slightly different range such as *jam**, *jam*°, etc. The multiple-range theory hopes to combine the advantages of classical logic with those of a semantic account of vagueness. But in comparison to epistemicism, it seems that it aggravates its main problems. For one thing, it exacerbates the problem of what determines sharp boundaries by multiplying them in the form of multiple ranges. For another, it also exacerbates the problem of the mismatch between the speaker's knowledge of meaning and the ascribed meaning (Are you aware of the numerous ranges of "jam"?). Ironically, Raffman subscribes to the idea that the "rules are discoverable largely by [...] self-reflective procedures" (2014, p. 22).

The mother of alternative logics for vagueness is three-valued logic. According to it, statements about borderline cases should receive a third truth-value, different from truth and falsity. However, while such a proposal might work for pure cases of indeterminacy where no tug-of-war along a similarity dimension is involved (such as Fine's nice numbers), it does not do justice to vagueness, and to higher-order vagueness. In effect, it replaces one sharp boundary (between "true" and "false") with two sharp boundaries ("true" – "neither true nor false", "neither true nor false" – "false". But the borderline cases are not a sharply delimited group. Moreover, to treat "neither true nor false" as a full-fledged truth-value that occurs in truth-tables and inferences does not reflect ordinary use. There simply is no third truth-value in the practice of ordinary language; there is not even a name for it. A popular criticism of the three-valued approach claims that it has problems with penumbral connections, statements that concern borderline cases. The idea is that "Either this fruit spread is a jam or it is not" is an instance of a law of logic and should be true in all cases; but according to three-valued logic it is neither true nor false in some cases. Now, I do not think that this is a problem at all for the three-valued approach. I discuss penumbral connections in the context of speech-act theory, and argue that negated instances of logical laws can be used to express that there is an unclarity (see section 8.5.

Nevertheless, concerns about penumbral connections were a driving motivation for applying supervaluationism to vagueness (Fine 1975). Supervaluationism starts from a three-valued assignment of truth-values to statements, and then introduces what is called "precisifications". They are two-valued, and have to fulfil certain constraints (for

Figure 2.: The three layers of supervaluationism:
From bottom to top: initial truth-assignment, precisifications, and super-truth
Light Grey = true, black = false, dark grey = neither true nor false
Left column = clear cases of not-F, right column = clear cases of F, middle column = borderline cases

instance about penumbral connections) in order to match ordinary language. In each of these precisifications, there is a sharp boundary. However, their multiplicity makes clear why speakers are reluctant to locate a sharp boundary (Raffman's multiple-range theory uses the same rationale). In a third step, one supervaluates over the precisifications. If all admissible precisifications of a statement are true, the statement is super-true; if all are false, it is super-false; if they are mixed, it is neither true nor false. We end up with a three-layered cake (figure 2).

But the problems of higher-order vagueness and the mismatch with actual linguistic practice regarding the third truth-value persist. Keefe (2000, p. 161) tries to solve higher-order vagueness by claiming that the notion of an "admissible interpretation" is itself vague. In that way, sharp boundaries between "true" and "neither true nor false" could be avoided. However, she does so by interposing a further category: borderline cases of being "true" or "neither true nor false". This category has again sharp boundaries. Since the problem recurs, further categories with sharp boundaries have to be interposed. Similar to three-valued logic, sharp boundaries are multiplied, though on a much larger scale. Moreover, a multitude of categories are introduced. Both have the effect of precisifying the meaning of vague concepts such as *jam* beyond recognition. A further problem relates to the notion of super-truth and to the standing of the precisifications. Since the goal is to solve the sorites paradox and give an account of vagueness that captures the meaning of the words of ordinary language, it would be anathema to regard them as a *reform* of ordinary language. But then, again, an aspect of ordinary language is postulated of which ordinary speakers have no idea that it exists.

A final major attempt to improve on three-valued logic is degree theory (Goguen 1969, Machina 1972, Lakoff 1973). Instead of three truth-values, it proposes an infinity of such

values. This is supposed to deal with higher-order vagueness: There is no sharp boundary between "true" and "neither true nor false". The downside is that, similar to Keefe's strategy, it leads to a great number of categories and sharp boundaries. As Sainsbury (1997, p. 255) remarks: "You do not improve a bad idea by iterating it." Similar to Keefe's proposal it suffers from massive over-precision; it ascribes more precision to an ordinary concept that is actually there. Is it true to a degree of 0.77359903 that the jar of banana jam is empty? It also incurs the problem of the mismatch with the practice of ordinary language. If ordinary practice does not indicate the presence of a third truth-value, it does so even less for an infinity of truth-values. The *prima facie* evidence there is applies to other categories. Half-truths usually refer to biased, incomplete stories, the verdict "partially true" to whole accounts, not single statements.

Contextualism cuts across the distinction between defenders and detractors of classical logic. It is the idea that blurred boundaries are a mere appearance, brought about by the circumstance that unnoticed changes in context never let the sharp boundary come into view. The main problem for this proposal is that the customary contexts such as comparison classes or contrasting cases can be specified, and that expressions with a fully specified context are susceptible to the sorites. Contextualism therefore needs a special sort of context that cannot be specified. Soames (1999) treats vague expressions as indexical; Raffman (1994) proposes to view the psychological state of the speaker as such a context; for Fara (2009), the context is determined by the interest of the speaker. But I see no reason why such contexts cannot be made explicit. Therefore, the contextualist strategy has little appeal for a solution of the sorites paradox. However, any explicable context may play a role to avoid occurring borderline cases in speech (see section 8.1).

On the whole, these theories of vagueness suffer from three major problems: An unsatisfactory account of higher-order vagueness, over-precision (which manifests itself for epistemicism in the mysterious relation between meaning and use), and an analysis of ordinary language which includes features of which speakers should be aware but are not.

This book is written within a framework that is based on Wittgenstein's later philosophy, and more broadly speaking on Ordinary Language Philosophy. It differs from the analytic framework of the main theories of vagueness in two broad ways: the comparison of language with a game, and a method that places great emphasis on understanding *what* is said and singling out nonsense. These differences enable a fresh and promising perspective on the problems of vagueness.

First, following the later Wittgenstein, this framework regards speaking a language as

a rule-governed activity; and the meaning of a word as being constituted by the rules for its application. Crucially, these rules may leave certain cases indeterminate; for certain cases, there may be no rules that regulate them decisively. After all, any natural language has grown into its role of serving the various communicative needs of human beings. It is no surprise then that it neither regulates abnormal cases nor infrequent cases of little practical value. This attitude towards studying language is expressed by Wittgenstein in the following way:

> Wir können sagen: Untersuchen wir die Sprache auf ihre Regeln hin. Hat sie dort und da keine Regeln, so ist *das* das Resultat unsrer Untersuchung. (*BT*, p. 254)

In contrast to the attempts to capture a vague language by a variety of precise and completely determinate systems, this opens the possibility to describe ordinary language in a way that does not lack exactness, but which leaves a place for genuine indeterminacy.

Second, the framework includes a method that promises to resolve philosophical debates by paying close attention to the actual use of words, and by showing that certain attempts to say something fail and are nonsensical. However, whether the appeal to nonsense is effective in the case of the sorites paradox is an open question of this project. It is possible that the analysis of the actual use of words has other modes to bring to bear on problems as well.

Apart from these aspects of my framework relevant to vagueness, the methodology of OLP I subscribe to is committed to further central ideas against which objections have been raised. For one thing, for OLP, philosophy involves *a priori* reasoning, and the *a priori* is elucidated as (a rule-based) analyticity. Against analyticity, Quine objects that it cannot be reduced to non-semantic notions. However, his argument presupposes that one subscribes to his austere programme of reducing semantic to non-semantic notions, and thereby presupposes that semantic notions are problematic (Quine 1951, Strawson/H. P. Grice 1956, Glock 2003a, ch. 3). Williamson objects against analyticity that there are counterexamples to the idea that understanding an analytic statement is to endorse it. But Williamson's purported counterexamples either amount to a *different* understanding or a misunderstanding of the sentence that is, in one understanding, an analytic statement (Williamson 2006, Schroeder 2009). For another, for OLP, the use of an expression is constitutive for its meaning. Grice's category of conversational implicatures (P. Grice 1989) shows that not all features of use are constitutive for meaning. A charge based on

Grice's insight then claims that OLP mistakes features of the use of words for features of their meaning. However, this charge does not work as a general objection, but only as an objection against particular analyses of the meaning of a word. Even if some OLP analyses turn out to be false, their method may still be valid (Glock 1996b, Hanfling 2000, ch. 10, Baz 2012, ch. 1). Indeed, Grice's category should be part of the arsenal of analysis of any OLP philosopher.[1]

Given that the conventional proposals regarding vagueness are problematic, and that I bring a distinctly different framework to bear, the following project suggested itself: Does my version of a Wittgensteinian framework afford a means to resolve the problems of vagueness? Thereby, both the Wittgensteinian framework needs to be explored (in relation to vagueness, but also concerning the mentioned differences), and applied to resolve these problems. Part I attempts to get a clearer view on Wittgenstein's method, Part II reconstructs Wittgenstein's views on vagueness, and Part III attempts to resolve the core problem of vagueness, the sorites paradox, by implementing this framework.

Part I engages with the methodological considerations of the *Investigations* (mostly in §§ 89–133), with a focus on issues of interpretation and systematic appropriation. A popular, if perhaps simplified, view of Wittgenstein's method has it that philosophical theories are demonstrated to be devoid of sense by resorting to ordinary use. I take issue with three aspects of that simplified view. First, for him, a philosophical investigation is centred around problems, and persons who have those problems, not theories. Second, the demonstration of nonsense is only one mode of many to relate an utterance made in a philosophical investigation to the use of words. This is especially relevant for the sorites. Third, it is a crucial question what "ordinary use" means here. A better way to bring use into play is by reference to the actual use of words of the relevant speaker, or group of speakers.

Chapter (1) provides a rough sketch of what I see as Wittgenstein's method, which includes his view of the aim of a philosophical investigation as resolving philosophical problems, and of the role of aetiology, i. e. of tackling the sources of why a particular person has a particular philosophical problem. Then, it tackles the interpretative challenges of Schulte and Conant who deny that Wittgenstein had a method. Schulte claims that Wittgenstein's method is little more than a skill, and thus not a method in any ambitious sense of that word. Concerning Schulte, I argue that his requirements for

[1] Hanfling (2000) and Baz (2012) are recent attempts to defend and apply the methodology of OLP.

ambitious methods are either misguided, or fulfilled. Conant's view is that the philosophy of the later Wittgenstein involves not *one* method, but a variety of methods. Regarding Conant, I criticise his argument that Wittgenstein's acknowledgement that there are a variety of grammatical forms implies that there are a variety of methods. Moreover, I elucidate a concept of *method* which allows Wittgenstein to combine an *overarching* method with a variety of problem-orientated methods.

Chapter (2) addresses the issue of the role that ordinary use can play for a Wittgensteinian method. An extreme interpretation of his method sees Wittgensteinian philosophers as invariably demonstrating that a certain claim is nonsense by comparing it to ordinary use. Part of this view is a criterial conception of nonsense, where deviation from ordinary use is seen as nonsense. I argue that none of the four discussed elucidations of "ordinary use" can serve in such a criterial conception of nonsense. Neither statistically prevalent use, nor a normatively declared standard use, nor ordinary use as the privation of metaphysical use, nor the actual use of a group of speakers can serve such a role. More advisable is an internal conception of nonsense, where explanatory gaps and inconsistencies in the actual use of the relevant speaker or group of speakers are a characteristic of nonsense. Moreover, an immanent critique of the use of a group of speakers allows for other modes of critique as well. Most importantly, a speaker's description of his own use may not hit the mark, as is the case in the sorites.

Part II investigates what Wittgenstein says about vagueness, and related topics. As the framework of this book is based on the later Wittgenstein's philosophy, the focus lies on the *Investigations*. However, this project faces the problem that the meaning of "vagueness" in the *Investigations* is less than clear, and that it is discussed in the context of his earlier work, the *Tractatus*. The *Tractatus* in turn is even more aphoristic and condensed. In order to engage with the *Tractatus* on the topic of vagueness, it was necessary to study Frege, who greatly influenced the early Wittgenstein. The study of Frege, however, has its own merits, as he is the initiator of the modern discussion of vagueness, and the origin of the metaphor of unsharp boundaries.This is particularly important in order to appreciate the terminological difference for "unsharp boundary" between early and contemporary Analytic Philosophy (see section 3.2).

On a terminological note, the chapters on Frege and the *Tractatus* differ from the rest of the book in that in them, I speak of propositions instead of statements. For one thing, this is customary, for instance in the translations of the relevant works. For another, it marks subtle differences in speaking about language. Frege and the *Tractatus* antedate

speech act theory with its emphasis on a variety of speech acts, and the idea that the same sign can be used to perform different speech acts. For Frege, a sentence (*Satz*) expresses its sense, which I call "proposition", which belongs to the Fregean third realm of abstract objects. For the *Tractatus*, a proposition (*Satz*) is a symbol, that is a sign together with its sense (cf. Glock 1996a, p. 315). That is, a *Satz* is once a mere sign, and once a sign plus sense; and a proposition is once thought to be an abstract object with ontological undertones, and once without (cf. *TLP*, 4.0312). In contrast, I follow Strawson in speaking of "statements" wherever I am interested in declarative speech acts. A sentence can both be a token, that is, a physical sign, or a type, that what such signs have in common. Statements are on a level in between, they are a case of Strawsonian uses. Two occurrences of different tokens of the same type may have the same use (e. g. when someone explains to friend A: "Montpellier lies in France", and then to friend B). Accordingly, for Strawson, there is a distinction between type-meaning and uses of sentences, and the latter can be infelicitous. Frege and the *Tractatus* operate without the benefit of that distinction, and are liable to conflate the two levels.

In chapter (3) I contrast Frege's use of "concept with unsharp boundaries" with the modern "vagueness". The main contrast is that Frege is concerned with indeterminacy *per se*, and the modern discussion only with a subkind of it, namely indeterminacy in a transition zone. Moreover, I reconstruct Frege's commitment to nihilism in the light of his ideal language philosophy. Because a concept-word is not truth-apt if it is unsharp, and because most words of ordinary language are vague in the modern sense, most statements made with an ordinary language are, for Frege, without a truth-value. Finally, I consider a number of alternative interpretations of Frege, notably Burge's epistemicism, and Puryear's claim that Frege had an interest in a non-logical point of view. I argue that they have either a limited scope, or are not convincing as an interpretation of Frege.

Where Frege thought that ordinary language is logically flawed, the *Tractatus* turns this position on its head. The demands of logic need to be fulfilled if thought be possible, and ordinary language fulfils them. Chapter (4) provides an interpretation of the *Tractarian* position on determinacy. It tries to elucidate the central and enigmatic remark "Die Forderung der Möglichkeit der einfachen Zeichen ist die Forderung der Bestimmtheit des Sinns" in the light of Fregean indeterminacy and some passages of the *Notebooks*. In a first step, I discuss the *Tractarian* motivations for deep-level analysis and an ontology of simple, indestructible objects. In particular, the picture theory together with the demand for determinacy lead to the view that a completely determinate proposition

needs to be analysable in terms of simple names. In a second step, I explore the formulations of determinacy of the early Wittgenstein, who takes from Frege the demand for determinacy of concepts, but reformulates it for propositions. Finally, I argue for an interpretation of the *Tractatus* which characterises its position as an epistimicist one, where ordinary language is completely devoid of any indeterminacy, even though the completely determinate meaning of a proposition is unknown to its speaker.

Wittgenstein's work can be divided into roughly three parts. The early phase consists mainly of the *Tractatus* and the associated notes and can be said to last from 1913 to 1922, according to the writing and publication dates of the documents. The middle phase lasts from 1929 to 1936, when Wittgenstein started to work on what is now § 1-188 of the *Investigations*. The late period then can be located between 1936 and 1951, until Wittgenstein's death. The middle phase can again be roughly divided in an early middle phase from 1929 to 1933, when the *Big Typeskript* was completed, and a later middle phase from 1933 to 1936 (cf. Schulte 2009, p. 467).

Chapter (5) presents the results of my research into Wittgenstein's *Nachlass* concerning vagueness. As it turns out, those passages that introduce something new in relation to the *Tractatus* and the *Investigations* can be found in the early middle phase, in particular in the *Philosophical Remarks* and the *Big Typeskript*. The *Remarks* endorses a primary, phenomenal language (a commitment that is undercut by the later private language argument), and claims that blurredness is an intrinsic feature of experience, understood phenomenally. It then explores its logical features under the name of visual space. The *Big Typeskript* mentions the problem of the heap (though not the sorites paradox), discusses the adverb "roughly", and considers a psychological experiment for "seen length". Moreover, it claims that the order "Make the smallest heap" is nonsense, and discusses a related, sharp concept for which the same holds.

The topic of vagueness in the *Investigations* is easy to neglect. It is discussed explicitly only in a few condensed remarks, and is eclipsed by other, related topics such as family resemblance. Chapter (6) tries to reconstruct Wittgenstein's views on vagueness in the *Investigations*, where directly and indirectly related topics from §§ 1–242 are relevant. Wittgenstein is not concerned with the sorites paradox but takes up Frege's worry that an indeterminate concept is no concept at all. This absence is natural, if, as I argue, family resemblance concepts need not be sorites-susceptible. Moreover, in the parlance of rules, a determinate concept is one which is "everywhere bounded by rules" (*PI* § 68). Abnormal cases such as a dis- and reappearing chair provide a first stumbling block for

such a view. Against Frege's worry, Wittgenstein argues that blurred concepts are not completely useless, and that drawing a sharp boundary *changes* the concept. Moreover, the claim that a logical approach is better because it is more exact is open to the charge that the value of exactness is goal-dependent, and that it is unclear what goal such an exact language serves. Finally, I reconstruct an argument against the conceivability of a completely determinate language that uses considerations that are closely related to the topic of rule-following.

Part III attempts to solve the sorites paradox and give an account of modern vagueness and indeterminacy within a Wittgensteinian framework. Chapter (7) regards the sorites paradox from the perspective that speaking a language is a rule-governed activity, where the rules may leave things open, and cases indeterminate. I argue that the sorites paradox presents itself as a dilemma concerning Tolerance: Either we accept it, maintain unsharp boundaries and are led into paradox; or we deny it, and end up with sharp boundaries that misrepresent our vague concepts. But the distinction between rule-formulations and empirical statements allows for a third possibility: Tolerance is not a rule. I first argue that the status of Tolerance is indeed that of a potential rule, not of an empirical statement (or other statuses). Then, I distinguish between internal and external negation, and argue that Tolerance is false due to an external negation (because it is not a rule), which does not imply sharp boundaries. Finally, based on a Wittgensteinian account of what it is to follow a rule, I discuss a variety of scenarios concerning the boundaries of concepts, and conclude that Tolerance is not a rule that ordinary speakers follow. Accordingly, the sorites paradox is a fallacy because Tolerance is not a rule, which negates it externally, and does not imply sharp boundaries.

This provides the basics of the not-a-rule approach. By looking closer, however, more problems crop up, in particular concerning the nature of borderline cases, and what it is for a concept to have blurred boundaries. Chapter (8) brings Austin's speech act theory (1975) and Strawson's notion of presupposition (1950) to bear on borderline cases in speech, and discusses blurred boundaries in the context of a detailed and comprehensive perspective. I characterise borderline cases as cases where the rules of language do not provide a decisive ruling, and where there are reasons both to call a case F and to call it not-F. In other words, the type-meaning of vague concepts leaves the correct application of that concept indeterminate in borderline cases. However, this indeterminacy of the type-level does not translate into straightforward indeterminacy on the level of statements and other speech acts. For it is infelicitous to make speech acts that are unclear, and

occurrent borderline cases are such a case. In that case, the statement has a truth-value gap (in a sense which excludes it from inferences and truth-tables). But in many cases, speakers can avoid occurrent borderline cases, if only by *ad hoc rulings*, and the borderline cases of type-meaning have no consequences for speech. A further case is when the unclarity of the type-meaning is problematised; negated instances of, for instance, Bivalence can be used for that purpose.

The problematicity of the sorites paradox in the formulation with the Tolerance principle can be intensified by the conditional version, which uses a large number, say ten thousand, of individual conditionals such as "If this fruit spread contains ten crystals of pectin and is a jam, then it will also be a jam with eleven crystals." Applying the external negation in the form of "P is not a rule" here provides only half the rent. For such individual conditionals *can* be used as a criticism. Against this, I argue that the conditional version forces us into a comprehensive and detailed perspective, which is itself problematic because it requires answers that by necessity distort our ordinary concepts. This comprehensive and detailed perspective is also in play in higher-order vagueness and the forced-march sorites. The key to higher-order vagueness is to recognise the irreducibility of vagueness. An appropriate semantics of a vague word needs itself to be vague in order to avoid a clumsy approximation by a number of full-fledged categories. Finally, I take a look at the attractions of the sorites, which include the fact that conditionals have a place in our practice, that a generalisation to a comprehensive and detailed perspective is natural, that the role of rules in our language is little understood, and that the desire for clarity and exactness is liable to overreach.

I

Methodological Considerations

1

Does Wittgenstein have a Method?

The Challenges of Conant and Schulte

Wittgenstein's methodological considerations belong to his most important contributions to philosophy. These considerations suggest that there is *a method* which informs the work of the later Wittgenstein. That claim, however, has been challenged in two ways. Schulte (2002) calls into doubt the idea that the later Wittgenstein had a method in any ambitious sense of the word. For Schulte, Wittgenstein did at best have a method in a non-ambitious sense, which is little more than the skill of dealing with philosophical problems. Schulte gives two reasons: The first is that Wittgenstein has not provided an illuminating description of his method at any stage of his thinking. The second is that there is no theory in the framework of Wittgenstein's philosophy, and that methods in some way require theories. Conant (2012) maintains that, for the later Wittgenstein, there is not one method, but a multitude of methods. Conant provides two reasons. The first is his reading of (*PI*, § 133), which states that "there is not a single philosophical method". The second is his insistence that a plurality of grammars require a plurality of methods. The aim of this essay is exegetical and I will defend three interpretative claims against the challenges of Schulte and Conant. First, Wittgenstein did *employ* a method that provides general guidelines for practising philosophy; second, he *did reflect* on his method, which is embodied in his methodological remarks; third, what he takes to be his method does not diverge substantially from his employment of it.

To show what is at issue, let me give a brief description of my interpretation of Wittgenstein's method: Philosophy is concerned with questions of sense, and not with questions

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of truth. A philosophical problem is a misunderstanding or a lack of understanding of the meaning of some words.[1] Their meaning is closely connected to the use that relevant speakers make of these words (sometimes, the same sign has a different use for different groups of speakers or even individual speakers). However, the explanations of the meaning of words which a person gives cannot be accepted without further ado; they might incorporate philosophical mistakes and misunderstandings and are the *raw material* for philosophy. Nevertheless, the use of words provides the *basis* for deciding whether there is a philosophical problem, a misunderstanding of the grammar of language, and what a correct understanding amounts to. A person can be aware that something is amiss, resulting in disquietude, or she might be unaware. In the latter case, her use can be problematised by an interlocutor.[2]

The use of words as *basis for philosophical investigations* is one of five central elements of Wittgenstein's method. The other four are the nature of *philosophical problems*, what the *aim* of philosophy is, and the *sources* of problems as well as the *means* to resolve them. A philosophical problem is a misunderstanding of the use of words. The *aim* is to attain complete clarity, that is, a completely clear understanding of the use of the relevant words, in such a way that the initial misunderstanding is resolved. Complete clarity is not the same as comprehensiveness, the idea that all potential problems need to be solved. In that way, philosophy is closely tied to individual persons. We cannot resolve philosophical problems once and for all, but have to deal with a philosophical problem in relation to the person who has it. Moreover, there is a further diagnostically relevant category: the *sources* of philosophical problems. The diverse means, methods or procedures used to dispel the misunderstanding must not only take into account the philosophical problem, but also the sources of the problem. Thus, different means may be employed for the same philosophical problem, if the latter has more than one source.

Some of the sources that Wittgenstein mentions are: the craving for generality (*BBB*, p. 17), the search for definitions where no definition is possible (*PI*, § 65f), being held captive by a picture (*PI*, § 115), the lack of surveyability (*PI*, § 122), the tendency of

[1] This does not imply that all misunderstandings are philosophical problems; let alone that every lack of understanding is a philosophical problem. For instance, many misunderstandings due to phonetic similarities are not philosophical. But in every philosophical investigation, a flaw in the understanding is at issue – be it an actual misunderstanding, or a lack of understanding that is problematised by a prudent person *before* she has fallen into misunderstanding.

[2] There are, of course, different interpretations, for instance Hilmy 1987, Glock 1991, Baker 2004b, Baker/Hacker 2005b, K. Morris 2007, Kuusela 2008 or Horwich 2012.

interpreting a use of a word as a strange process (*PI*, § 196), a diet of too few examples (*PI*, § 593), and a misleading surface grammar (*PI*, § 664). Further, once the sources of a problem are singled out, the philosopher may choose the *means* or methods most appropriate to deal with these sources and with the problem. Some of the means that Wittgenstein employs or recommends are: inventing language games (*PI*, § 2, 48), comparing the grammar of two words (*PI*, § 78) or substituting one form of expression for another (*PI*, § 90) (for a longer list, see section 1.5).

What is the sense of *method* used in my affirmation that Wittgenstein has "a method of philosophy"? He has an *overarching* method. That is a method which guides the invention and the application of particular methods or procedures to particular philosophical problems. A short note on my terminology: Since these particular procedures may also be called "methods", I don't distinguish between methods and procedures but between overarching and problem-oriented methods. (Procedures are problem-oriented methods.) Problem-oriented methods are *means* for the resolution of particular philosophical problems. The overarching method is a general strategy for how to deal with new problems: Determine what the philosophical problem is by looking at the use that a person makes of words (see whether what she says makes sense), check for sources of the problem and try out some means (without guarantee of success) to reach the aim of complete clarity.

Importantly, the method that I ascribe to Wittgenstein is not foundational. It is not the case that the method is the axiomatic foundation of philosophy, and once we get the method right, all philosophical problems will be resolved in due course. In that case, the method would become an unjustifiable monolith. This is not the role that Wittgenstein accords to his method. For him, questions about method are an integral part of philosophy – discussing them is itself nothing else than doing philosophy (cf. *PI*, § 121). Nevertheless, his philosophy and his method are not without presuppositions (cf. Glock 2007a, p. 59). Problems and questions about them are to be treated like any other philosophical question. This implies that Wittgenstein's method is vulnerable to attacks on its own presuppositions. If it turns out that *meaning* has no connection whatsoever to *use*, his method loses its appeal.

I begin by telling two different stories about the development of Wittgenstein's thought in the 1930's (section 1.1). Then I consider Schulte's position and state my reasons for not being convinced by it (section 1.2). In section (1.3), I trace a strict dichotomy of two kinds of methods as a source for Schulte's refusal to acknowledge that Wittgenstein had a method and provide an alternative taxonomy of methods. Next, I discuss Conant's

position and argue against his claim that grammars require methods (section 1.4). Finally, I turn to perhaps the most salient objection to my position: the interpretative challenge which is posed by § 133 of the *Investigations* (section 1.5).

1.1. Continuity vs. discontinuity

A *prima facie* difficulty for interpretations that deny that Wittgenstein had a method (Schulte and Conant) is that from 1930 up to 1938 there are many passages about method in the *Nachlass*. There are remarks in which he speaks about his method (TS 225, p. 2, *MWL*, p. 113), in which he ascribes central importance to it (*LWL*, p. 21) or in which he, in succinct form, states what his method is: "Methode der Philosophie: Die übersichtliche Darstellung der grammatischen Tatsachen" (*BT / TS 213*, p. 414) and "My method throughout is to point out mistakes in language." (*AWL*, p. 27) There is an obvious line of defence, which both Schulte and Conant take: They claim that Wittgenstein's views about method changed between the early and the late 30s. In a rough sense, we can separate a middle from a later Wittgenstein: The middle Wittgenstein was under the illusion that his way of doing philosophy was guided by *one method* and ascribed central importance to it. The later Wittgenstein avoided that illusion and, with the exception of (*PI*, § 133), stopped talking about method. Schulte and Conant give different accounts of the illusion. For Schulte, the middle Wittgenstein wrongly compared chemistry with philosophy in respect of method and thus ends up with demands that his method cannot meet (2002, p. 404–406). Conant, on the other hand, sees a close connection between grammar and method: A plurality of grammatical forms requires a plurality of methods. The middle Wittgenstein's insistence on both a plurality of grammatical forms and on there being one method brings tension into his philosophy (Conant 2012, p. 633). I will comment briefly on two of the most striking passages in the work of the middle Wittgenstein. The first is taken from lecture notes from 1930:

> The nimbus of philosophy has been lost. For we now have a method of doing philosophy, and can speak of *skilful* philosophers. Compare the difference between alchemy and chemistry; chemistry has a method and we can speak of skilful chemists. But once a method has been found the opportunities for the expression of personality are correspondingly restricted. (*LWL*, p. 21)

In this passage, Wittgenstein intimates that philosophy has undergone a great change, brought about by himself, and specifies the difference as one of doing philosophy *without* a method versus doing philosophy *with* a method. The introduction of a method in philosophy brings sense to speaking of skilful philosophers. Thereby, the philosophers lose their nimbus; they are only skilfully applying a method, not expressing their personality in their work.

The second passage is from an early version of the preface of the *PI*. In this version, there is an additional passage which did not make it into the final version:

> Ich beginne diese Veröffentlichung mit dem Fragment meines letzten Versuchs, meine philosophischen Gedanken in eine Reihe zu ordnen. Dies Fragment hat vielleicht den Vorzug, verhältnismässig leicht einen Begriff von meiner Methode vermitteln zu können. (TS 225, p. 2, cited after Schulte 2002)

The fragment to which Wittgenstein refers in this passage could either be the whole body of the text of the *Frühfassung* (TS 220 & 221), or the first half of it (TS 220), or a segment of TS 220 (cf. Schulte 2002, p. 400). The later omission of this passage is relevant to my discussion, because it apparently removes the only allusion to the method from the preface, which, in turn, reduces the occurrence of the word "method" to a few paragraphs of the body of the *PI*.

Turning the tables, Schulte and Conant now demand an explanation from interpreters who claim that Wittgenstein has a method: Why are such passages absent or even omitted from the *PI*? Here is my explanation: Broadly speaking, there is a shift from middle Wittgenstein's emphasis on the discovery of the one method to his refraining from emphasising such a discovery later on. This, however, does not mean that the later Wittgenstein does not have *a method*. I see one main reason for the shift: Wittgenstein's continuous concerns about his own vanity. Proclaiming that one has found "the method of philosophy", that, after 2000 years of history, one has finally brought philosophy finally onto the right path, even if true, is a claim that is easily seen as demonstrating vanity. Accordingly, it is replaced by a more humble one: "A method is now demonstrated by examples" (*PI*, § 133; see section 1.5). Whether this method can sustain the claim that it is the right one depends, among others, on its ability to solve philosophical problems.

More importantly, the shift isn't as substantial as one might think. The later Wittgenstein may have a method without saying that he has one. The possibility exists that the remarks in the *Investigations* that concern methodology amount to a method. If

they fulfil our requirements for being a method, then there is no obstacle to identifying a method in Wittgenstein's later work. That of course depends on the notion of *method*, which will be discussed in section (1.3).

At this stage, both readings have a story that is *prima facie* plausible. However, the account that Schulte and Conant give of the alleged illusion of which middle Wittgenstein was a victim has to be scrutinised. They think, in their respective versions, that Wittgenstein had good reasons to give up the talk of "the method of philosophy". As I will show below, the reasons that they present are not convincing.

1.2. Schulte: "Little more than a skill"

Schulte's main contention is that the method which plays such an important role for middle Wittgenstein is in fact "little more than the skill itself" (2002, p. 405), or even that it is "a method basically amounting to a special type of skill" (2002, p. 406). The core passage to which Schulte alludes is the one cited above (*LWL*, p. 21). There, Wittgenstein compares two sets of differences: On the one hand, the difference between a philosophy that has a method and a philosophy that has none, and, on the other hand, the difference between alchemy and chemistry. The way Schulte reads this passage is very peculiar. On the one hand, he takes the cue "skill" from it, in order to claim that Wittgenstein's philosophy has no ambitious method (see below), but that what can be learned from Wittgenstein is a special type of skill. On the other hand, he claims that in that very passage, Wittgenstein is the victim of a "self-misunderstanding of his method" (Schulte 2002, p. 406).

The picture that Schulte draws is the following: A skill requires a framework for its exercise. For chemistry, the framework includes a theory. Since, for Wittgenstein, philosophy has nothing to do with scientific or quasi-scientific theories, the framework of philosophy does not contain a theory. Rather, it is "at most a kind of tradition, an institution or a common background." (Schulte 2002, p. 404) Now, at least in the quoted passage, Wittgenstein draws a sharp contrast between traditional philosophy and a new way of doing philosophy (like the contrast between alchemy and chemistry). So there is a new element, which does not belong to tradition, the old institution or the common background. Schulte concedes that "Wittgenstein felt that he had *discovered* something," and he compares this discovery to the discovery of "a [new] style of painting, a method of playing the violin, or a new form of cuisine. No one will expect you to be able to

give an informative account of that sort of style or method." Schulte then concludes: "His failure to produce anything like a satisfactory account of this method would have made it clear from the beginning that to the extent he really had found a new method it was a method of the second kind, that is, a method basically amounting to a special type of skill." (Schulte 2002, p. 405) In effect, Schulte sees a mismatch between middle Wittgenstein's *description* of his method and his practice: That the latter is not guided by an ambitious method, as the description would have it.[3]

Schulte distinguishes between two kinds of method. Those methods which are a mere skill (or not much more) and those which are (considerably) more than a mere skill. The latter are called "method[s] in an ambitious sense" (p. 406), and the former we may call a methods in a non-ambitious sense. Note that it is not *per se* clear how "method in an ambitious sense" (henceforth "ambitious method") has to be understood. What Schulte does not have in mind are methods with ambitious goals. For Schulte, there may be non-ambitious methods with ambitious goals. If Wittgenstein conceived of his method as one that could be used to resolve all philosophical problems, his goals are certainly ambitious. But, according to Schulte, his method is not of the ambitious sort. Thus, "ambitious method" is best understood as being defined as more than a mere skill.

There is an issue concerning the notion of *skill* as it is used by Schulte. Do skills belong to the qualities of a person? In that case, we could say: "He is generous, arrogant, tall, and has an amazing skill as a sculptor." If so, there is tension with the standard explanation of "method" as a way of doing things (by OED, Macmillan and Merriam-Webster). As such, a method is not a quality of a person, and it becomes questionable how non-ambitious methods are methods at all. That is why Schulte's "skill" has to be understood as "craft": An art, profession, occupation or trade that involves a skill. The sentence "The craft of cabinetmaking was much admired in colonial times" refers to a trade or profession, and not to the skill employed in that trade. Then, "method in the non-ambitious sense" is compatible with the above notion of *method*: Do it in the way of goldsmiths, cabinetmakers, or philosophers.

What does it mean for a method to be a mere skill, to be a non-ambitious type of method? Based on the picture above, we can isolate two features. i) The application of non-ambitious methods is not connected to a theory. ii) Non-ambitious methods do not

[3] Ironically, Dummett (1978, p. 434, 452) saw a similar mismatch in the work of the later Wittgenstein, though in the opposite direction: That his practice includes theses even though his description suggests otherwise.

require an illuminating description to be taught and passed on. Accordingly, in Schulte's picture, we find two arguments why middle Wittgenstein was wrong in thinking that he had a method in an ambitious sense, and why the later Wittgenstein does not have such a method:

I Wittgenstein's philosophy doesn't have a theory in its framework, therefore it has no method in the ambitious sense in its framework.

II Wittgenstein didn't give an informative account of his method, therefore he has no method in the ambitious sense.

Ad I: With which sense of theory does this argument work? Following Hanfling (2004, p. 189), two senses of "theory" might be relevant here. The first are causal-hypothetical theories, as they are common in empirical sciences, such as the postulation of an unobserved body which explains the movement of visible planets and comets. If Schulte uses "theory" in that sense, then all non-naturalistic philosophy is excluded from having a method or methodology. That, of course, would be a harsh consequence. The second sense might be called quasi-scientific. Unlike the first kind, such theories do not offer causal explanations. Similar to it, they try to explain (in a non-causal sense) phenomena by postulating "entities or processes that do not lie open to view" (Hanfling 2004, p. 189). In this second sense, Schulte's requirement is tailor-made to exclude Wittgenstein's philosophy from having a method. Postulate hidden entities or you will have no theory and therefore no method. But then, Schulte's requirement for an ambitious method to have a theory is itself problematic. Wittgenstein's method is excluded from being a method on the grounds of one of its own features: its special status regarding theories.

Ad II: I take it that the alleged absence of an illuminating description is not sufficient proof that a particular method is of the non-ambitious kind. Rather, it is circumstantial evidence to that effect. Accordingly, there can be an illuminating description of a particular method even if the method is non-ambitious. I disagree because the required circumstantial evidence is missing. In my opinion, Wittgenstein's reflections on his method are of great interest and constitute an illuminating description. To sketch my reasons: In the *Investigations*, there are many remarks that contain informative descriptions of Wittgenstein's method, most prominently in the so called "philosophy chapter" (§ 89–133), as well as in solitary remarks such as (§§ 52, 254–255, 309, 314, 340, 371, 387, 464, 599, 664). Indeed, these passages are difficult to interpret due to the "nature of the investigation" (*PI*, pref.), Wittgenstein's laconic predilections and the

different voices of the work. But for instance in (§§ 89 and 90), he provides a sketch of the kind of investigation a philosophical investigation is. It neither studies facts of nature nor causal connections, but is a grammatical investigation that studies "the *kinds of statement* that we make about phenomena." (§ 90) Granted, some concepts require clarification: For instance, what is "everyday use"? (§ 116) But this shows only that Wittgenstein has not resolved *every* philosophical problem related to his method. It does not show that he has not given an illuminating description of it in any sense. The framework that Wittgenstein provides with the description of his method is not "at most a kind of tradition, an institution or a common background," (Schulte 2002, p. 405) but an account of the nature of the philosophical problems and their solution.

1.3. Remarks on the concept of *method*

One source of Schulte's denial that Wittgenstein had a method is that Schulte retains a strict dichotomy between two kinds of method. The first kind is a method that is basically a special type of skill (and thus a "method" in square quotes). The only alternative Schulte mentions is a method which consists of "rules [...] that spell out what a player should in certain situations do in order to win." (Schulte 2002, p. 403) Schulte's category of rules conductive to winning should be construed broadly: A physiotherapist does not win against his patient, but he can successfully ease the other's pain. And there might be rules for him to do so. In the case of philosophy, Schulte envisages rules like:

1) If your opponent believes in substances, you are well-advised to begin by pointing out to him that not all nouns function according to the pattern exemplified by labels that may be stuck onto the objects they fit. (Schulte 2002, p. 403)

That is, Schulte demands from an ambitious method in philosophy that it has explicit instructions for *particular cases*. For all of them? For many? For just a few? Anyhow, the *Investigations* doesn't supply such explicit if-then rules. Then, according to Schulte's dichotomy, there remains only one kind of method which could be ascribed to Wittgenstein: a method as a skill. Perhaps one can restate Schulte's two kinds of methods in terms of rules. This restatement exceeds the literal phrasing in Schulte's paper, but may be helpful in presenting a related view: An ambitious method should consist of a comprehensive set of rules that can be applied to any particular philosophical problem. A non-ambitious method would typically be taught by engaging with particular cases,

whereby at best some such particular rules may be mentioned (although the particular philosophical investigations could be conducted without invoking any rules). I disagree with that picture. To begin with, the requirement of comprehensiveness, which is in force only for the ambitious kind of method, differs depending on whether the set of philosophical problems is logically fixed or open. In the first case, a list of particular rules could be comprehensive. However, it is not clear that any participant of the debate would like to claim that the number of philosophical problems is fixed. In the second case, comprehensiveness can only be achieved if the rules do not apply to particular philosophical problems, but merely give more general guidance. This brings me to the disagreement. Rules of a method don't need to be applicable to particular cases. On the contrary, they can apply to classes of cases, or even to all relevant cases. The rules that I will mention at the end of this section are of this kind. That philosophical problems are to be solved by studying the use of words is such a general rule. Moreover, once general, methodological rules are acknowledged, a method can achieve comprehensiveness not by solving every problem in advance, but by providing rules that outline a way for solving every problem (that is to be solved by that method).

In what sense of "method" does Wittgenstein have a method? As an alternative to Schulte's two kinds of methods, we can distinguish methods according to three cross-cutting distinctions: sequential and non-sequential arrangement of rules; creative and mechanic application of rules; overarching and problem-oriented methods. Here are some definitions:

Sequential method: has an ordered and finite set of rules.
Non-sequential method: its rules are not ordered into a series or it has an unlimited number of rules.
Creative application of rules: there is leeway in the application of rules.
Mechanic application of rules: rules are applied without (much) leeway.
Overarching method: used to create methods that deal with problems.
Problem-oriented method: used to deal with problems.

I will consider a few examples which combine these features in various ways. First, the method of picking a lock by using a bump key: Insert the bump key and pull it back until you notice a click. Then, turn the key in the right direction while hitting it with a special hammer. Repeat until the lock opens. Of course, in order to follow this quick method you need to have the right instruments, and you need to practice. Nevertheless, once you

have learned, you can *mechanically* apply the steps in the right order *(sequential)* and you will be able to open many locks *(problem-oriented)*. Some other examples of methods with the same features are: a method of folding a t-shirt in two seconds, a method of projecting a geometrical figure and a particular method of salary negotiation the main point of which is to avoid naming a number. Since these methods are both *mechanical* and *sequential*, we could call them *algorithmic methods*. These are not algorithms since they operate not exclusively with signs or functions (Turing machines would have a hard time tying a knot). But analogous to algorithms, they can solve a given range of problems by applying a finite set of mechanically applicable rules.

Second, formal logic provides a method of checking arguments for their validity (*problem-oriented*). Part of this method is to formalise arguments of natural language. Normally, when we do that we don't try any proposed translation blindly but go with the most plausible one, only to change it later if trouble comes up. That is, the rules of translation are not ordered in a sequence (*non-sequential*). (It's an open question whether one could construct an algorithm which performs these tasks.) It's *creative*, since the choice of formalisation depends on our latitude of judgement.

Third, take Descartes' method in his *Discours de la Méthode*, p. 33. It consists only of four rules that are to guide human understanding in *all* its endeavours. One could argue that the rules are ordered in a *sequence*: First, get rid of all supposed truths if there remains some doubt. Second, divide the problems into subproblems. Third, start with those problems that are easy to solve. Fourth, strive for completeness in your enumeration of knowledge. These rules are to be applied *creatively*. For instance, we have to judge ourselves where to carve out the sub-problems. Finally, while the method does not call explicitly for the creation of problem-oriented methods, the extreme generality of the method implies that some further methods need to be applied, for instance to check whether a supposed knowledge is free from any doubt. Thus, Descartes' method seems to be *overarching*.

Fourth, there is a method of dog training called "dog whispering". Instead of viewing the relationship between owner and dog as one of issuing commands and following them, it sees it as one of mutual understanding, and thereby sets the goal for dog training. Consequently, what is perceived as a problem is not only the behaviour of the dog who chews on the cushions while the owner is absent, but also the behaviour of the owner who comes home, gets angry, swears in a raised voice at the dog and then misinterprets the ensuing fear of the dog as guilt. A means to resolve such problematic behaviour, then, is

to both change the behaviour of the dog through conditioning *and* and the behaviour of the owner through explanation. Naturally, these guidelines alone cannot teach one to be a skilful dog whisperer. But they provide guide rails to integrate solutions of particular problems into a wider framework, adapt existing procedures to new cases or even invent new procedures. Thus, this method is *overarching*, *non-sequential* and *creative*.

Any method must delimit the range of problems it deals with and state what counts as a solution. For some methods, this can be done very easily. The problems of the bump-key method are locked doors (and other locked things) and the solution of the problem consists in an unlocked and open door. For logic, the problems are obvious: arguments. But the solution is less clear-cut than an open door – has the argument been misrepresented or is it really not valid? The range of Descartes' method is less general than it first appears: It is limited to *knowledge*. And rule one states what amounts to a solution: No reason for doubt is left.

Many rules of the methods mentioned above state what has to be done in order to succeed, in order to reach a given goal: They are regulative. These rules are also constitutive in a certain aspect: They determine what counts as following the method *m*. Nevertheless, such rules normally do not determine what counts as an activity for which they provide guidance. For instance, the rules of the angling method of fishing do not determine what fishing is. In this respect, Wittgenstein's method is a special case, because it is strongly linked to a conception of philosophy. For instance, the rules that state the aim and the problems of the method *determine* the goal, instead of giving guidance how to reach the goal. They are therefore constitutive. But constitutive rules can be action-guiding, too. If there is a mismatch between the goal as we pursue it in practice and our real need, a change of the pursued aim in our practice is crucial to achieve what we really need to achieve. The belief that it is a constitutive rule of chess that the queen can only be moved while the player stands on his head might prevent some players from moving the queen. Likewise, a difference in the conception of philosophy corresponds to a difference in how the subject is pursued, for instance in the role that is ascribed to scientific discoveries. If there are good reasons for or against one of the conceptions or methods, then this discussion is of obvious importance. Thus we can see how the five elements which I identified in Wittgenstein's method are action-guiding. The rules that determine the nature of problems and aims are important to pursue aims that address our real need (cf. *PI*, § 108). But what are the rules of Wittgenstein's method? Here is a very inadequate attempt to press the five elements I have mentioned into rules. They are at a high level of abstraction and would require further elucidation.

R1 A philosophical problem is a misunderstanding or a lack of understanding of our use of words.

R2 The aim of philosophy is to attain a completely clear understanding of the use of those words that are problematic.

R3 What counts as a misunderstanding and what counts as an understanding is to be determined by studying our use of words.

R4 One philosophical problem may have different sources.

R5 A variety of methods have to be applied depending on the problem, its sources and the other problems that an individual has.

1.4. Conant: "Methods, not a method"

Conant claims that for the later Wittgenstein, there is not *one* method, but a multitude of methods. Obviously, this alludes to the last and boxed sentence of *PI*, § 133 (cf. Conant 2012, p. 635). I will discuss the interpretative challenge of this sentence in the next section. Conant (2012) provides an intricate story of the development of Wittgenstein's thought from early to middle to later Wittgenstein in terms of method and grammar. Moreover, Conant is of the opinion that these two, grammar and method, are linked in an important way. One can put it succinctly: *Grammars require methods*. The general story (Conant 2012, p. 640, 642) is that early Wittgenstein held that there is one method of philosophy and that there is a fixed number of possible forms of grammar. Middle Wittgenstein realised that the possible forms of grammar are not fixed, but clung to the idea that there was one method. Later Wittgenstein then realised that the idea of one method is undermined by there being an unlimited variety of forms of grammar, and consequently abandoned the idea that he had one method. I will point out some gaps in the story. But let's first see how Conant puts it:

> One might formulate the negative aspect of the insight which underlies the shift here in question as follows: the relinquishing of the first of these definite articles (*the* logic of our language) requires the correlative abandonment of the second of these definite articles (*the* method of philosophy). One can also formulate the point here at issue in more positive terms as follows: an open-ended, infinitely extendable conception of a family of possible forms of grammar comes to be seen to require a correlatively open-ended, infinitely

> extendable conception of a family of possible forms of philosophical method. That the one requires the other, however, takes time and work for Middle Wittgenstein to come to appreciate. (Conant 2012, p. 640–641)

Some elucidations are in order. First, what does "open-ended, infinitely extendable" mean? I understand this as alluding to a potential infinity: On the one hand, it is obviously not the case that we have an infinite number of elements at hand. On the other hand, the set of elements is unlimited in the sense that we can always find a new element. The antonym of it is "limited": At a given point, we have written down all elements of the set. This point may have not been reached yet, but it is in principle attainable. Consequently, Conant does not contrast one method with a plurality of methods, but with an unlimited plurality. Second, what are "forms of grammar"? I take them to be equivalent to "grammatical forms". Of course, early Wittgenstein did not talk about "grammar", but about "logic". Thus, are we actually comparing "logical forms" of early Wittgenstein with "grammatical forms" of middle and later Wittgenstein?

How can the shift away from *the* logic towards a plurality of logics (or grammars) be understood? A false start would be to say that the *Tractatus* would advocate a limited number of logical forms which middle Wittgenstein replaced by an unlimited number of grammatical forms. The problem is that this is not true of the *Tractatus*: For one thing, there is an important distinction between logic and the *application* of logic (*TLP*, 5.557). Only the latter decides which elementary propositions there are, and with that, which logical forms there are. Logic does not decide whether there is a 27-termed relation (*TLP*, 5.5541). For another, the "logical forms are *without* number" (*TLP*, 4.128). That is, in the *Tractatus*, the logical forms are unlimited, too.

A better way to tell Conant's story takes into account the distinction between three different levels in the *Tractatus*: On the first level is the general form of a proposition. All propositions with a sense conform to this one form: They are truth-functions of elementary propositions. To deny this yields an alternative position like i) that there are more than one general logical forms, but still a limited amount (one could call them "categories"), or ii) that there is an unlimited number of general logical forms. Since on this second alternative, the distinction between general grammatical forms and grammatical forms becomes doubtful, on that account, the first level collapses into the second. The second level comprises the logical forms: proper names, predicates, 2-termed relations, 27-termed relations. Thus, ignoring depth analysis, "is a boy" and "is a man" have the same logical form, but not "is an uncle". For the latter is a disguised relation.

The third level is then the logical form of a specific sentence which, in the terminology of the *Tractatus*, is called the "logical structure".

For Conant, the contrast between the early notion of logic and the later notion of grammar is pertinent; that is, the move from *logic* to *grammars*. So here is my take on Conant's story. Middle Wittgenstein abandoned the idea that all propositions conform to *one* form, the general form of a proposition. But he retained the other two levels in modified form. Instead of logical forms we have now grammatical forms, which also include differences such as between "has a key" and "has a pain". Thus, there is indeed an important change in terms of grammar or logic: The early Wittgenstein thought that there is *one* logic of language, which is topic-neutral, and can be used to talk about any topic. Middle Wittgenstein, however, realised that the grammar of language can take very different general forms in different language games. The story now explains that the one general logical form corresponded to one method, but that this monism of method had to be given up once the monism of general propositional form had been given up.

Third, what are "forms of method"? Perhaps we can understand this expression by analogy with "forms of grammar". A form of grammar is what is common to several sentence-types. For instance, "Romeo loves Juliet" and "Peter beats Paul" have the same form (in one respect) as in both sentences, a two-termed relation occurs. Analogously, in the piecemeal processes of tackling confusions of two persons, what is common to two applications of method is a form of method. For instance, one philosophical move is to point to a difference of symbol of two signs that appear to be similar (cf. Conant 2012, p. 630, *TLP*, 3.23). That is, forms of method turn out to be nothing else than *methods* in what I call the problem-oriented sense. Thus, the substitution method and the elimination method for solving mathematical equations are two methods, or two forms of method.

The story rests on an insight on Wittgenstein's part. This insight can be presented in the form of an argument:

2) In philosophy, an unlimited number of general forms of grammar requires an unlimited number of methods.[4] (*grammars require methods*)
3) The number of general forms of grammar of our language is unlimited.

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4) Hence, philosophy requires an unlimited number of methods.

[4] In other words: If our language has an unlimited number of general forms of grammar, then philosophy should have an unlimited number of methods.

Middle Wittgenstein realises (3) contra early Wittgenstein, but fails to grasp (2). Only the later Wittgenstein makes the next step and reaches the conclusion (4). The argument is a valid *modus ponens*, but I see two problems with it.

On a preliminary note, it would be uncharitable to read this argument as excluding any approach to philosophy which does not accord a central role to forms of grammar or logic (Kant, Husserl, Descartes) from having a method at all. After all, Conant's formulations are directed at Wittgenstein's development, not at the history of philosophy in general. Moreover, he could react by generalising premise (2) to any "object or medium of philosophical investigation (logic / grammar)" (Conant 2012, p. 640). That is, if the object of philosophical investigation sports an unlimited number of forms, then an unlimited number of methods is required to deal with them.

The first problem is the main premise (2). Conant is clearly committed to something like this by his story and the long quote above. But why should we accept anything like that? For one thing, there is no indication that Wittgenstein thought that a particular grammatical form must be investigated by a particular method. The methods that early and later Wittgenstein provide are not *grammar-dependent*. For the *Tractatus*, two methods come to mind: First, the method of constructing an ideal notation (cf. *TLP*, 3.325, 4.1121), and second, the method of using this ideal notation to demonstrate that certain sentences are nonsensical, and to understand other sentences clearly (*TLP*, 4.112, 6.53). Perhaps one could argue that Wittgenstein's ideal notation requires a general propositional form. Accordingly, the methods of the *Tractatus* would depend on it. Even if so, this shows only that the *Tractarian* methods have to be discarded together with the general propositional form. But it does not show anything about the methods of the *Investigations*, especially not that there is an unlimited number of them.

What about the *Investigations*? If Conant is right, methods that deal with a particular form of grammar should be all over the place. Some of the means that Wittgenstein employs are: inventing language games (*PI*, § 2, 48), looking how a word is taught (*PI*, § 9, 35, 179), comparing the grammar of two words (*PI*, § 78), substituting one form of expression for another (*PI*, § 90), a surveyable representation (*PI*, § 122),[5] imagining different general facts of nature (*PI*, § 142, *PPF*, § 366), showing something about the grammar of one word by connecting it to the grammar of another (*PI*, § 492) and looking

[5] There are different ways to interpret "surveyable representation" in the *Investigations*. Kuusela sees it as a sort of umbrella term for all methods that Wittgenstein uses (Kuusela 2008, p. 269). Glock favours a more specific interpretation (1996, p. 280). I tend towards a specific interpretation different from Glock. But this matter need not be decided here. Independent of the interpretation, surveyable representations are an important means that Wittgenstein advocates.

at different contexts of the use of a word (*PI*, § 593). In my opinion, none of these procedures or methods are *grammar-dependent*. Some of descriptions of them directly use the word "grammar" – but, of course, comparing the grammar of two words in no way restricts this procedure to any *particular* form of grammar.

Moreover, that some methods may be applied to different forms of grammar is especially virulent for methods that deal with the obstacles of a proper understanding, with the *sources* of confusions. For instance, in the *Big Typescript* (p. 406), Wittgenstein claims that philosophical problems are not difficult for the intellect, but for the will. This requires strategies or methods to outwit the will: We have to fight certain intellectual temptations, tendencies. Similarly, an important obstacle to properly understand that part of grammar that causes trouble are ideals, pictures of how it must be which have settled in the mind. Just to present a clear view of the relevant grammar may here not achieve the desired effect. The ideal has to be tackled separately, for instance by constructing a simple language game as in (§ 2, 48). That is, there are many procedures that are *grammar-independent*. But then, even if Conant were right in claiming that the later Wittgenstein rejected the idea of the early and middle Wittgenstein that philosophy had *one* method, that he did so in virtue of abandoning the view of *the* grammar is highly implausible. For the relation between grammars and methods is much too feeble to sustain such a major change.

The second problem concerns the claim that an unlimited number of methods is incompatible with there being *one* method. Initially, this seems plausible, since we have to understand the latter claim as there being *at most* one method. Explaining away the tension is essentially the same as dealing with the interpretative challenge of the last and boxed sentence of (*PI*, § 133). I will address this in the next section.

1.5. "Method" in the *Investigations*

In what way does the term "method" occur in the *Investigations*? Concerning method, we can distinguish three levels. On the first, there is an overarching method that guides the creation of new problem-oriented methods: This is the one of which I gave a short five-rule description in section (1.3). On the second level, there are various problem-oriented methods such as imagining a language that would fit a particular philosophical theory or picture (and others mentioned in the last section). The third level comprises applications of problem-oriented methods; that is the methods used in actual or exemplary

philosophical investigations. Here, it is important to keep in mind that philosophy treats problems of understanding. And it is *individual persons* who misunderstand the use of words. That is, this level consists in addressing the confusions of particular persons; the means or methods applied need to take into consideration the sources of the problems and the other problems the person has.

The word "method" does only occur in a few remarks in the *Investigations*. One use concerns the connection between sign or picture and meaning, and whether there is a method of projection to get from one to the other (*PI*, § 139, 141, 366, 548). Another use is as a method of measurement (*PI*, § 242). Both don't deal with the sense of "method" that interests us. Closer to our interest are two remarks that concern a *problem-oriented* method. (§ 48) mentions "the method of § 2", and applies it to a new case. The method, in this case, is to consider a language game where an antecedent description of language as a whole is indeed correct. The point of studying "the phenomena of language in primitive kinds of use in which one can clearly survey the purpose and functioning of words," is to "disperse[...] the fog" which has been brought about by a "general concept of meaning" (§ 5). That is, it is a method or procedure that is destined not to tackle a particular form of grammar, but a grammar-independent hindrance for taking the grammar of language for what it actually is: diverse. This is important insofar as it shows that Wittgenstein used "method" in a sense which presupposes that there are several methods. But this of course does not rule out the possibility that he used that word in another sense, too. The most pertinent remark about method is (§ 133) which would merit a discussion of its own. For the sake of brevity, I will only to examine the two occurrences of that word.

> Sondern es wird nun an Beispielen eine Methode gezeigt und die Reihe dieser Beispiele kann man abbrechen. (*PI*, § 133)

"Examples" is written in plural, "method" in singular. Is this the last surviving reference to Wittgenstein's *one* method? Or is he talking about one *of many* methods? If "eine" were in italics, the former reading should be favoured. But since it is not, both readings can be upheld. I lean towards the former, since it chimes with my over-all reading. Instead of laying the foundations of philosophy, what we get from the later Wittgenstein are a bunch of examples of philosophical problems plus their resolutions. In these philosophical investigations, a method has been at work, and, Wittgenstein insinuates, the method can be learned by considering the examples. Importantly, however,

"showing the method" does not exclude making methodological remarks, of which (§ 133) itself is a prime example.

> Es gibt nicht *eine* Methode der Philosophie, wohl aber gibt es Methoden, gleichsam verschiedene Therapien. (*PI*, § 133)

Here, it seems that Wittgenstein explicitly denies that philosophy has *one* method. My response to this is that in the *Investigations*, it is crucial to notice deliberate ambiguities, such as that of "being hidden" (*PI*, § 91, 126 vs. § 129) or the use of one word in positive and negative contexts, e. g. "essence" (*PI*, § 89, 371 vs. § 92, 97, 113, 116). Likewise, there is a deliberate ambiguity between the method that is "demonstrated by examples" (*PI*, § 133c3) and the insistence that there are *many* methods (*PI*, § 133box). The first is about the *overarching* method, which provides a general strategy to tackle philosophical confusions and an understanding of the nature of philosophical investigations. The second is about is about the variety of procedures, means or problem-oriented methods that the method enables to create or apply. While Wittgenstein did not draw this distinction himself, he is committed to it. The reason for this is that it removes the tension to the passage in the preface where he talks about the "nature of the investigation". Thus, the sentence in the box attacks the misconception that the talk of "a method" implies that there is one *fixed* procedure which one can follow – the method consists, on the contrary, of applying and creating a variety of problem-oriented methods. It is poly-procedural.

The claim that Wittgenstein in the *Investigations* held that he had a method to teach is supported by a passage from the preface. While, indeed, the word "method" has vanished completely from the final version of the preface, there is a remark about the "nature of the investigation":

> Und dies hing freilich mit der Natur der Untersuchung zusammen. Sie nämlich zwingt uns, ein weites Gedankengebiet, kreuz und quer, nach allen Richtungen hin zu durchreisen. [...] So ist dieses Buch eigentlich nur ein Album [von Landschaftsskizzen]. (*PI*, pref.)

And the sense of "nature of investigation" is not a lightweight one. For it has direct repercussions for how one can go about the investigation. There is no natural, antecedently established order which a presentation of Wittgenstein's philosophy could follow. Rather, those philosophical confusions and problems that are in force in our dialogue partner

or in ourselves need to be tackled. Moreover, treatment of the problems leads to "long and meandering journeys" ("langen und verwickelten Fahrten") (*PI*, pref.). "Verwickelt" carries with it the idea that the problems are interconnected, and the relations between them weave a chaotic and asymmetrical net. Several strands of connection go out for instance from the referential doctrine of meaning, which can stand at the beginning of many philosophical problems – disregard for language use, or the idea of mental entities.

Now, we can go back to the question of continuity or discontinuity between a middle Wittgenstein who explicitly claimed to have found a method and the later Wittgenstein who does not talk much about method. As we have seen, the reasons that Schulte and Conant put forth to explain why Wittgenstein had to change his opinion turn out to be weak. But then, there is no good story of discontinuity. Thus, the only plausible story we have is one of continuity: The later Wittgenstein became more cautious in advertising his method as a central element of his philosophy, but continued to reflect on and describe his method. In sum, there is good evidence that the later Wittgenstein had a method in an ambitious sense, one which he tried to pass on through his samples of philosophical investigations and the methodological remarks that accompany them.

Conclusion

Schulte's claim that Wittgenstein's method is little more than a skill in effect states that no method can be ascribed to the middle and later Wittgensteins. But Wittgenstein's remarks on how to pursue philosophy constitute just that: a method. The philosophical importance of Wittgenstein's work is not exhausted by his contributions to particular areas of philosophy such as philosophy of mind, language or mathematics, nor by the alleged exemplary status of his philosophical investigations. His reflections on how to engage with philosophical problems are a contribution to philosophical method with which contemporary philosophers can profitably and critically engage. Moreover, Schulte's two requirements for a method are problematic. The first one, that methods are connected to theories, excludes Wittgenstein's method from being a method in virtue of one of its own features *qua* method. The other requirement is reasonable, but fulfilled: The claim that Wittgenstein does *not* provide an informative description is unconvincing. The comparison of the philosophical method with a way to play the violin leaves the method as something magical, which is only acceptable if we are not ourselves to play the violin.

Conant is quite right to stress the diversity of methods in the *Philosophical Invest-*

igations. However, the distinction between overarching and problem-oriented methods paves the way for having the cake *and* eating it. The overarching method, which is the subject of methodological discussions, guides the invention and application of the various problem-oriented methods. Moreover, the idea that methods are closely tied to general grammatical forms is misguided. Not a single method which I have come across in Wittgenstein's writings is directed *at a particular form of grammar*. It would be interesting to see *one* example of such a method. Conant, however, needs a whole batch of them to validate his claim that there is an important connection between methods and grammatical forms.

What remains is the enormous groundwork of digging up and specifying the various methods used by Wittgenstein in the course of doing philosophy. Since this cannot be done without engaging with the particular philosophical topics to which these methods are applied, it is truly a Herculean task. Moreover, the distinction between problem-oriented and overarching methods could be applied to the *Tractatus*, too. Thus one could try to describe the method of the *Tractatus* independently of the one procedure of (6.53). Perhaps a discussion of the second method of the *Tractatus* (3.325, 4.1121) of studying ordinary language under the aspect of an ideal notation could bring to the fore the fundamental division between setting up the ideal notation and applying it.

2

Ordinary Use, Nonsense, and the Nature of Philosophy

One of the promises of Ordinary Language Philosophy is to solve philosophical problems by appeal to the ordinary use of expressions. For one thing, classical problems of conceptual analysis can be framed as scrutinising whether proposed definitions of an expression fit the ordinary use of that expression. For instance, in Plato's dialogue *Laches*, the general of the same name defines courage as to stand and fight in battle. And Socrates proceeds to show him that his definition is too narrow, because even Laches would *call* some troops "courageous" that are fighting while moving back (Platon 1994). According to the OLP story, Laches is in effect misdescribing his and other's use of that word. For another, following the *Tractatus* and Logical Positivism, Ordinary Language Philosophy has another philosophical move in their repertoire: Expose nonsense by, in the case of OLP, appealing to ordinary use. For instance, regarding the question "Could (philosophical) zombies exist?", an OL Philosopher might point out that, upon reflection, it does not make sense. Assume that "zombie" is explained as a being that acts in all respects just like human beings, but has no consciousness. Then, the OL philosopher might claim that, according to the ordinary practice of ascribing consciousness to beings, acting in the same way as human beings is sufficient for having consciousness. But then, it makes no sense to say that some being acts like a human being, but has no consciousness.

Given that ordinary use seems to play such an important role in the methodology of Ordinary Language Philosophy, two questions come to the fore: What is ordinary use? And what is its methodological role? In particular, there is a significant confusion about

what ordinary use is both for students and critics of Ordinary Language Philosophy (as can be seen by some of the objections that I will discuss). A confusion that leads to an oversimplification of the methodological role of ordinary use.

My primary interest in these questions is systematic. Can a contemporary Ordinary Language Philosophy come up with defensible methodology that ascribes a role to ordinary use? At the end, I think that such a methodology can be defended, but that it does not provide a simple way to decide philosophical discussions. Hard philosophical work is still needed. Historically, I will refer mostly to Ryle and Wittgenstein. While Wittgenstein is perhaps the greater inspiration for my philosophy, one can find greater methodological insight in Ryle than many would expect. In this regard, this essay is also a vindication of Ryle.

Of the two philosophical moves that I mentioned, conceptual analysis and exposing nonsense, I will focus on the second. The arguments that will be discussed for the mode of exposing nonsense are *mutatis mutandis* applicable to conceptual analysis. One difference is that exposing nonsense combined with a certain conception of ordinary use leads to a criterial conception of nonsense, for which there is no parallel in conceptual analysis.

CCN: Every use of words which accords to ordinary use makes sense, every use of words which deviates from it is nonsense.

This is a straw man's position, and a very extreme one. Nevertheless, it does capture a tendency in OLP to overplay the nonsense card, and do it in ways that lack a proper justification. Neither Ryle nor Wittgenstein endorse this received view of OLP. However, perhaps one can find exponents of OLP that are at least committed to the criterial conception of nonsense. Malcolm comes dangerously close in *Moore and Ordinary Language*.

My argument proceeds by considering a number of conceptions of ordinary use in combination with the criterial conception of nonsense, and showing that these combinations are not tenable. The main point then is that the criterial conception of nonsense is not a tenable view of the methodological role of ordinary use. As an alternative methodological role, I then consider ordinary use as the basis for an immanent critique. Nevertheless, the lessons about the conception of ordinary use can be transferred to the new methodological role.

Before we can get started, however, two preliminaries have to be dealt with. To start, the name "OLP" has suggested to some that OLP is concerned with expressions of

ordinary language (e. g. "chair", "raven", "knowledge" or "meaning") as opposed to technical expressions (e. g. "proton", "walking catfish", "cognition", "pragmatics" or "language game"). Similarly, one and the same sequence of signs may have a lay and a scientific use, for instance "empty space". Accordingly, even OLP figures could be blamed for using vocabulary that does not belong to ordinary language. But this is not at all what OLP is doing, as Ryle makes clear: It is the ordinary use of *any* expression, be it common or technical, which is of importance for OLP (Ryle 1953, p. 167–8).

The second preliminary is what aspirations are held for the method of OLP. In a historical overview over OLP, Martinich writes about the method of OLP:

> The method [of OLP] is to use features of certain words in ordinary or non-philosophical contexts as an aid to doing philosophy. (Martinich 1998)

The aspirations of this OLP method are very low. It merely aims to be an aid in doing philosophy. In that way, the role of ordinary use would be an auxiliary one as well. But the central insight of Wittgenstein and OLP – which needs to be elaborated – seems to ascribe a more central role to ordinary use. Here is a bold take of the aspirations of ordinary language philosophy.

OLP-Aspirations: All philosophical problems can be solved by the OLP method.

This is of course a sweeping claim. It is not entirely clear that any proponent of OLP in the wide sense is committed to it. The early Wittgenstein comes closest when he declares that "the problems have in essentials been finally solved" (*TLP*, pref.). The later Wittgenstein comes to a similar view by reflecting on the nature of philosophical problems (*PI*, § 123). At any rate, whether this programmatic claim is true is not important for our present purposes. It serves to set a high goal. If it turns out that there are particular philosophical problems, or philosophical problems of a certain kind that evade solution by the OLP method, this would in itself be a highly interesting finding without thereby undermining the whole project of OLP. A popular area of doubt is ethics and political philosophy, where for instance McDowell (2009, p. 367) is among the skeptics.

The starting point for my discussion is a straw man's position that goes well beyond what actual proponents of OLP have endorsed. Nevertheless, it brings questions about the role of ordinary use and the place of nonsense in the method into sharp relief. In particular, by trying to find a conception of use that can fulfil a criterial role, important

objections will be discussed, and the role of the use of words in philosophy clarified. In section (2.1), I discuss the proposals to regard ordinary use as the statistically prevalent use, and as a normative standard. The first of these has to contend with the armchair objection, the second with the normative rebel objection, and the semantic inertia objection. In section (2.2), I take a look at how Wittgenstein explains ordinary use, at Baker's attempt to characterise ordinary use as the privation of metaphysical use, and at a characterisation of ordinary use as actual use. I then argue that none of these proposals can serve a criterial conception of nonsense. Because all five attempts fail, I then regard the criterial conception of nonsense as failed. Section (2.3) portrays an alternative conception of nonsense, embedded in the procedure of immanent critique. Finally, section (2.4) deals with an objection against this alternative conception of method. In order to evade the normative rebel objection, I propose to regard the relevant use of words, for methodological purposes, as potentially individualistic. But this may be seen as raising the question whether there are any shared philosophical problems.

2.1. Prevalent and standard use

According to to the criterial conception of nonsense, there is a distinction between ordinary use and non-ordinary use where the former is logically in order and the latter is logically flawed. In the wake of the logical positivists' anti-metaphysical stance, the non-ordinary use is often called "metaphysical use". Accordingly, there is a pejorative sense of "metaphysical use". In this sense, it becomes absolutely crucial for *CCN* to be able to answer Rorty's question: "How can we distinguish the ordinary from the metaphysical use?" (cf. Rorty 1970, p. 13). I will discuss five proposals that give an account of what ordinary use is. All five proposals presuppose that speaking a language is a rule-governed (and normative) activity. They differ in the kind of description they give of the linguistic activity of a group of speakers. The first is:

PU: Ordinary use is prevalent usage, that is the body of rules that most speakers of a certain population follow.

Accordingly, the ordinary use of a community of speakers is determined by the relative frequencies of uses of words. Note that there is no straightforward way to decide which exact relative frequency counts as prevalence. Moreover, *PU* turns the task of identifying

ordinary use into an empirical task that is to be studied with the statistical methods of science. The results of such studies will be of the form:

1) In population P, 30 % of the speakers follow rule R.

Combined with the criterial conception, such a notion of ordinary use can be used to reject uses that do not accord to the standard set by the majority (of uses) as nonsense. Not a very inviting perspective! Russell comments in terms of a charge addressed at OL philosophers that he entitles "insincerity":

> What in fact they believe in is not common usage, as determined by mass observation, statistics, medians, standard deviations, and the rest of the apparatus. What they believe in is the usage of persons who have their amount of education, neither more nor less – less is illiteracy, more is pedantry – so we are given to understand. (Russell 1953, p. 304)

Adapted to our framework, we may call it the armchair objection: If *PU* holds, the armchair analysis that is favoured by OL philosophers will turn out to be i) not representative, and ii) methodologically hopelessly inadequate. If proponents of the criterial conception accept *PU*, they should take over the methods of empirical linguistics. But that of course would abolish the categorical difference between philosophy and empirical sciences, as favoured by OLP. Russell's charge of insincerity is insofar off the mark as, if any proponent of OLP subscribes to such a notion of ordinary use at all, the main figures of OLP certainly do not. For instance, Ryle is adamant that appeal to an "ordinary use" understood as a (or the most) widespread use in a society has no philosophical relevance. He writes:

> The reader considers the mode of employment that he has long since learned and feels strengthened, when told that big battalions are on his side. In fact, of course, this appeal to prevalence is philosophically pointless. (Ryle 1953, p. 177)

This settles the interpretative issue. But how does Ryle counter the armchair objection? Crucial here is Ryle's distinction between use and usage. A description of usage describes frequencies of rule-governed uses, and can be falsified by statistical methods. A description of use, however, remains on a normative level. It touches upon how to do something, and how to do it correctly.

> There cannot be a misusage... Learning it [the use of something] is learning how to do the thing; it is not finding out sociological generalities. (Ryle 1953, p. 174)

For Ryle, to learn a language is not to learn sociological generalities, but to learn a technique. In describing the rules of language, we then try to capture the rules of a technique that we know well from using the technique, but whose rules we don't carry around in a rule-book. To formulate those rules is then to make explicit what we know implicitly. But this is signally different from describing how many people use *those* rules, or how many people use an expression in which places.

Thus, Ryle can counter the armchair objection by making clear that he is not concerned with usage, with empirical descriptions of language-behaviour, but with the correct use of certain expressions. The proposal to identify ordinary use with prevalent use *(PU)* will play no role in the rest of the essay.

The next proposal takes up the theme of normativity and refers to standard use.

SU: Ordinary use is standard use (of a language), that is the body of predominant rules (of that language).

Here, we have an additional normative element, a socially sanctioned normativity. The predominant semantic (and other linguistic) rules of a society are set by an institution or a person that has the required power and influence. For English, there is no official regulatory body such as the *académie française* for French. Nevertheless, a few persons and institutions such as the authors of the Harvard Style Guide do have a huge influence on what counts as proper or standard English. And persons whose use does not conform to the rules of standard English will be sanctioned by society in a variety of ways.

Concerning Ryle, it is not completely clear where he stands in this debate. His denial of the effectiveness of big battalions may indicate that the is ready to consider, in the extreme, the use of an individual person, and thereby discard the philosophical pertinence of regulatory institutions such as the *académie française*. However, as far as I know, there is no reference to such cases. Nevertheless, when Ryle (1961, p. 227–8) writes that "The reproof 'You cannot say that and speak good French' is generically different from the reproof 'You cannot say that without absurdity'," he goes a long way towards disowning reference to standard use for philosophical purposes.

There are two main problems for *SU*. To begin with, it is the wrong kind of normativity for philosophical pertinence. Somebody who wants to get a job should write his cover letter according to the rules of standard use. But his failure to do so does not undermine the meaningfulness of what he writes, but only his chances of getting the job. Speakers can be held responsible for their normative commitments. But that presupposes that they are thus committed. Crucially, a speaker is only committed to those linguistic norms or rules of language that are in force for his linguistic behaviour, that he actually follows. Thus, if a speaker says: "He be working Tuesdays." he might be corrected for his improper English with reference to the standard rules of English. But he might reply that these rules are not in force for him. In fact, the example is taken from the African American sociolect, and there are good reasons to accept it as a proper dialect with its own rules. Of course, the scenario might repeat itself. An African American speaker might be corrected for not adhering to the rules of that sociolect, and he might equally reply that these rules are not the ones that he is following. This is the normative rebel objection. A consequence of it is that we cannot assume that the standard rules of English are in force for any given speaker, even those that we might want to classify as English speakers.

Of course, a speaker's challenge that a rule is not in force for him might ultimately turn out to be unsuccessful. Thus, a description of norms or rules may also be falsified. For instance, a speaker may challenge that the norm is in force for him, that the rule ascribed to him is not one which he follows. These issues will have to be resolved with recourse to normative interventions of the speaker: his explanations, justifications, corrections, etc., but also his applications of the expressions. The speaker cannot escape his own normative commitments. A slave trader, for instance, might agree that human beings should not be enslaved, but insist that the indigenous people he enslaves are not human beings. But if his explanations as to what human beings are do in fact apply to the indigenous people, he *should*, by the lights of his own explanations, call the indigenous people human beings. Nevertheless, the normative rebel objection shows that appeal to an authoritative standard of good or proper English has no traction in a philosophical debate. Normativity is involved, but the one we are interested in is the one the speaker wants to abide himself.

Another problem is semantic inertia. In a somewhat offhand mockery, Gregory (1984, p. 641) calls OL philosophers "guardians of semantic inertia". Behind this, there is a serious objection. Standard use is defined as the body of the predominant rules of proper

English. Of course, these rules change, and one might say that the institution which defines what proper English is may take account of changing usage by incorporating new rules. It might for instance record the new usage of "to refute" which does not require any argument and is synonymous with "to reject". However, even in that case, standard use lags behind how language is in fact used, and of course, used meaningfully. This conception of standard use, combined with the criterial conception of nonsense even claims that any new use not licensed by this body of rules is nonsensical. But then, there is a discrepancy between what is licensed standard use, and what makes sense. This, of course, is silly, since it excludes the actuality of linguistic change, the possibility of introducing a new use by stipulation, malapropisms, metaphorical uses, and other creative uses.

For illustration, consider the case of a new use that has no established place in the practices of a society or of an individual. It is in need of explanation. Sometimes such an explanation is given, as for instance by the stipulation that with "brounger", I mean "younger brother". Or take the case of a group of forest wardens. They have a large forest under their care and know all common tree diseases. One day, they encounter a tree that is not doing well, shows a yellow change of colour on its bark, but clearly cannot be classified as one of the known diseases. After encountering several other affected trees, they call it the "yellow bark disease". Then, depending on the seriousness of the situation, they might decide to cut down all affected trees. They can act immediately on their newly introduced expression.[1] Then, standard use does not only appeal to the wrong kind of normativity, but it also incurs a serious lag behind the realities of spoken languages.

2.2. Metaphysical and actual use

Neither prevalent nor standard use can serve as an elucidation of ordinary use when applied to a criterial conception of nonsense. A third proposal attempts to give the notion of ordinary use a defensible content by a close reading of Wittgenstein. Such an attempt takes its cue from the German adjectives "gewöhnlich" and "alltäglich". These are the words that Wittgenstein uses interchangeably when referring to ordinary use. The

[1] Like other terms in biology, the scientific classification of tree diseases might well be expert-dependent, such that the standard use of these expressions is laid down by experts, perhaps even by an official body. Nevertheless, the forest wardens don't need to wait for the expert body to introduce their own notion.

English word "everyday", just like the German "alltäglich", could be taken to insinuate that such things literally occur every day. But this is not the case. If somebody goes to church every Sunday, following the mass is not something that he does every day, but it belongs to his "everyday" life. Equally, even events that seem to be quite extraordinary, for instance floods, are a rather regular event in some regions (as in some mountain valleys). Indeed, in an early remark, Wittgenstein is prepared to count a variety of events such as birth, illness, death, sleep and dreams as being part of everyday (*Nachlass*, item 109, p. 200). The second word, "gewöhnlich", is closely related to "an etwas gewohnt sein". Thus, it expresses that the speaker is familiar with something. That is, this attempt identifies two features of ordinary use: regularity and familiarity. The problem of this proposal is obvious: What OL philosophers want to call metaphysical uses often have both of these features. While ordinary zombies have found their way into pop culture, metaphysical zombies have found their way into philosophical classrooms. Perhaps one could try to argue that we will never have the same familiarity with technical uses as with non-technical uses, but I don't think that this is a promising road.

The difficulty to explain that contrast, and to find an explanation of that contrast in Wittgenstein's work is highlighted by Baker's strategy. He wants to explain "everyday use" as the negation or privation of "metaphysical use". A forth proposal then combines the criterial conception of nonsense with Baker's strategy to explain the meaning of ordinary use. But it is highly problematic. Baker's negative characterisation is neither intended nor fit to bear the weight of a criterial conception of nonsense. He identifies metaphysical use by four features (Baker 2004a, p. 97–9).

a) Expressions of necessity and impossibility.
b) The use of words without antitheses.
c) Non-scientific statements having the form of scientific statements: simple formula as answers to "What is X?" questions.
d) Explanations grounded in the natures of things.

For one thing, these features are not problematic at all, as long as one distinguishes between expressions of semantic rules and empirical statements. For instance, "One plays patience by oneself" (*PI*, § 248) can be part of an explanation of what patience is, even if it expresses a necessity (a), leads to a use of words without antithesis (b) (one wouldn't play patience unless one plays by oneself), and is grounded in the nature of the game patience (d). For another, the combination of the criterial conception of nonsense with

Baker's account of ordinary use puts the weight of an entire philosophy on these four features. Strategically, this would be an inadvisable move because it requires acceptance of these four features as problematic (and as the only problematic ones). To meaningfully debate disagreements regarding their problematicity then would require to exceed the bounds of the method, thereby greatly hedging its aspirations.

Reflecting on how the use of words enters the philosophical investigations of Ordinary Language Philosophy one might come up with a fifth proposal. To describe the use of a word is to describe, in a well controlled manner, which rules the speakers are actually following. In the extreme case, "speakers" is replaced by "speaker". For we cannot assume that all speakers of a language follow in fact the same rules (armchair objection), or that they should follow the same rules (normative rebel objection). Then, one could attempt to identify ordinary use with actual use.

AU: Ordinary use is the actual use or practice of a group of speakers.

This is to say that what is important for philosophy are the rules that are actually in force for this group of speakers. It should not be confused with a distinction between potential and actual use, where the latter consists in those uses of words that have actually taken place, and the former in those uses of words where a speaker is asked or imagined to react to potential situations. If a speaker has applied "birlabul" only to red triangles, but we have no reaction for red squares, we may of course ask him in order to find out the rules that are actually in force for that speaker, which I here call "actual use".

Moreover, it is an idea that can be ascribed to Wittgenstein:

> Was ist an der Idee abstoßend, daß wir den Gebrauch eines Wortes studieren, Fehler in der Beschreibung dieses Gebrauchs aufzeigen, u. s. w. ? Vor allem fragt man sich: Wie könnte *das* uns so wichtig sein? Es kommt drauf an, ob man 'falsche Beschreibung' die nennt, die nicht mit dem sanktionierten Sprachgebrauch übereinstimmt, – oder die, die nicht mit der Praxis des Beschreibenden übereinstimmt. Nur im zweiten Fall ensteht ein philosophischer Konflikt. (*RPP I*, § 548)

What is a "false description of use?" Wittgenstein asks. The answer he contrasts the use of language that is sanctioned by society, that is, standard use, with the use "of the one who describes". In this case, the one who describes is the same person whose use *is*

described. That is, somebody describes his own use and goes wrong.[2] Crucial to our question is that Wittgenstein makes it clear that the philosophically relevant use is not standard use, but may be the use of one single person.

Are shared conventions indispensable to an account of language and meaning? What I have said so far does in no way decide the matter. The normative rebel objection opens the possibility that a speaker can object that a rule of standard use (of e. g. English) is in force for him, for what he has said. But this is compatible with the view that idiolectical rules are dependent on shared conventions, and thus with a communitarian view of language, as long as it allows that there are idiolectical rules.

However, this proposal cannot serve a criterial conception of nonsense. The problem is that utterances such as "There are several (philosophical) zombies at this conference" do follow a certain pattern; there are rules for the use of the word "zombie". While I do think that there are ways to show that there is nonsense involved, this does not hinge on the actuality of that use *per se*. There are rules for this metaphysical use, and some speakers in fact follow them (without noticing the trouble they get themselves into). There is even a distinction between a correct metaphysical use, and an incorrect one: "The square root of zombie is irrational." Of course, one may find creative and metaphorical interpretations of such utterances, but without further ado, this is an incorrect use of "zombie". When Wittgenstein talks about metaphysical use (*PI*, § 116) he concedes as much as that.

2.3. Philosophy as immanent critique

The criterial conception of nonsense in effect requires a characterisation of ordinary use that makes solving philosophical problems easy. Identify ordinary use, identify deviations from it, and the deal is done. But neither the characterisation of ordinary use as prevalent use, as standard use, as familiar use, as the privation of metaphysical use, or even as actual use can support such a conception of nonsense. The criterial conception of nonsense therefore is untenable.

Nevertheless, a possible way out is indicated by the consideration of actual use. While it too cannot serve as a criterion of nonsense, the argument of the essay at least shows that if the use of words is relevant to philosophy, it is the *actual* use that is relevant.

[2] It is indeed perfectly possible that somebody describes the use of somebody else. And it is also philosophically relevant, since if A criticises B's description of his own, B's use, as false, A describes B's use. And not his own, A's use.

Consider again conceptual analysis. How did Socrates make his point that courage is not to be equated with standing and fighting? *Not* with reference to prevalent use, nor to the standards of good Greek, but in relation to what Laches himself would call "courageous". Now, the actual use of a speaker is not just the rules that the speaker professes to follow. Rather, a potentially comprehensive analysis of what the speaker in question says and does is pertinent to identify the rules that the speaker actually follows.

Such a painstaking analysis amounts to what Glock (1991) calls an "immanent critique". A speaker's applications, and his normative interventions including explanations, justifications, and criticisms, and his overall behaviour are all pertinent for assessing whether he or she is victim of a philosophical confusion. This procedure is immanent, because the confusion is not demonstrated on the basis of any dogmatic assumption, but rather as an internal problem of what the speaker is saying and doing. One effect of this procedure is that the criterial conception of nonsense is replaced by an internal conception.

Internal Conception of Nonsense: An utterance fails to make sense if some words remain unexplained or there is an inconsistency in the actual use of the relevant group of speakers.

Here is a tentative (open ended) categorisation of philosophical problems, of internal problems in the actual use of a speaker or a group of speakers:[3] a) Some of the terms employed remain unexplained. For instance, one might summarise Wittgenstein's remarks on rule-following in that vein. Wittgenstein's interlocutor wants to maintain that the future (correct) use is determined *in a strange* way by the rule. But Wittgenstein then points out that all attempts to elucidate what is for a rule to determine future use in a strange way founder (Wittgenstein 2009b, §§ 138–242). Consequently, that phrase has no clear sense. Of course, to point out explanatory gaps is a standard philosophical move that antedates Wittgenstein or OLP. b) There is an inconsistency in the use of the subject. Here are again two cases: b1) The explanations contradict or offset each other. Let's say a zombie is a being that leads a life indistinguishable from a human life, and acts just like human beings, but has no consciousness. But if the grounds for ascribing consciousness to a being is that it leads a human life and acts like a human being, nothing

[3]This categorisation is influenced by Glock (1991, p. 83f). He oscillates between regarding this as a trilemma that befalls the introduction of *new* terms and regarding it as a categorisation of philosophical errors (FN 17). I opt for the second version.

is said by the use of "zombie". The two parts of the description of what a zombie is offset each other, ridding the question "Could zombies exist?" of a clear sense. b2) The explanations do not match the employment of the terms. The Sorites is a good example of this, of what Wittgenstein calls a "wrong description" (*RPP I*, § 598) of the use: While the Tolerance principle *seems* to be the correct description of our[4] use, it is not. c) They amount to a consistent use, but bypass the initial philosophical problem. Here are again two cases: c1) One subject combines two independent uses in her life. One is used when she is in her office, one is used when outside of it. As long as she does not conflate or confuse the two descriptions, there are two correct descriptions of her two uses. And here, Strawson's objection comes into play. The philosophical questions and problems are likely to be phrased in the use that is used outside of the office; therefore, setting up a consistent but different use does not address them (cf. *PI*, § 120; Strawson 1963, p. 513). c2) The subject not only has a consistent use, but has also given up completely the standard use of an expression. Depending on the extent of the difference to standard use, the ensuing pragmatic consequences are more or less severe. In the case of skeptics and nihilists, these consequences have in some cases a somewhat comical effect. Kratylos, who is convinced that one cannot say anything true, remains silent henceforth in his life. Skeptics about the senses have to be kept from walking over cliffs or into fires. Or skeptics about knowledge in general might give convoluted explanations when asked for the way to the restaurant: "Look, I suspect that the restaurant is that way, but I don't really know, because the city could have been rebuilt completely in the last five minutes since I passed there". And perhaps the problem can be accentuated. Not only what he says is relevant, but also how he acts on his beliefs. In what way is the doubt present if he goes himself to the restaurant?

After characterising ordinary use as actual use, what can be made of its antonym, "metaphysical use"? Both a pejorative and a complementary sense can be outlined. In the pejorative vein, one might call a technical-philosophical use of an expression that fails to connect to non-technical uses of other expressions a "metaphysical use". It would simply be a self-contained language game that may easily lead to misunderstandings because it uses terms such as "consciousness" that have a different use language games that are actually in use, too. Thus, somebody might claim that the question "Could

[4]This possessive is a source of trouble: to talk of "the use" would be unspecific, to talk of "the use of my interlocutor" would be unnecessarily over-specific. Of course, use tends to converge, but what is ultimately relevant is the use of my interlocutor (and mine).

there be (philosophical) zombies?" is a metaphysical use in the pejorative sense. Of course one can say that with justification only *after* having conducted a philosophical investigation. This sense can therefore serve in programmatic statements, or conclusions, but not in a move in a philosophical debate. In contrast, in a complimentary sense, one might call a technical-philosophical use that sums up an important part of the use of an expression a "metaphysical use": " 'Red' is a determinate of the determinable 'colour' "; " 'Language' is a family resemblance concept"; "Clutching one's cheek is a criterion and not an effect of toothache".

In a similar way, standard use, as defined in *SU*, still plays *some* role in the method, though again, not a justificatory one. For philosophical discussions are not cases of radical translation. That is, they occur against a background of an assumed shared understanding, which is provided by standard use. But it is of crucial importance to note differences from standard use. That is, in an alteration of Austin's phrase, the standard use of expressions is the first word, but it is not the last word. One function that the comparison with standard use can achieve is that it raises two questions: i) Why is that particular sign used albeit having a different meaning in standard use? and ii) How is the alternative use explained?

2.4. Can philosophical problems be shared?

The introduction of an individual-specific notion of use to deflect the normative rebel objection comes at a cost, though: it seems that the scope of philosophical problems and solutions has tightened considerably. In an extreme scenario, if a person has her own particular technique, if she uses most or all of her words in a way that nobody else does, then, most or all of her philosophical investigations are of no interest to her neighbour. If our use is the basis for a philosophical investigation, but "our use" turns into "my use", philosophy gets trapped in idiolects, one could think. Then, potential philosophical problems are particular to individual persons, and are not shared.

The current formulation of the charge is unclear in one respect: Is it about potential problems or about actual ones? Let me introduce some terminology. A potential philosophical problem in a primary sense is one that can be formulated with the extant resources of the language (a secondary sense will be introduced below). When the use of a person shows the typical tensions and inconsistencies of a particular philosophical problem, then that problem has become actual. A problem is virulent in a society if many

persons actually have that problem. Thus, if the charge is about actual problems, it is not a forceful one. For it would be a very strong position to claim that all human beings that exist (or have existed) do have the same actual philosophical problems. The charge is more interesting when formulated for potential problems. Then, to what extent two persons share the same potential problems depends on the actual differences of idiolects and on the translatability between them.

First, we may take a leaf out of Ryle's book who has the obvious and helpful rejoinder that there is a strong tendency to uniformity of use, not least because one purpose of using language is to be understood (1953, p. 177).[5] Thus, while the methodological emphasis is on the use of one person (be it myself or my interlocutor in a dialogue), the factual similarity between the use of different persons restricts the force of the charge. To put it crudely: If the use of language of two persons is similar to a large extent, then the idiolects will be similar to a large extent, too. And if the latter holds, there is a good chance that the philosophical problems that one person has in her idiolect can be formulated in the idiolect of the other. In that sense, the potential philosophical problems are shared between speakers with differing idiolects.

The charge of isolation may not only be formulated for idiolects, but also for languages: If they have different words with different uses, then the philosophical problems that can be formulated in French are particular to that language. Here, a common response can be given for both cases: One can *translate* between languages or idiolects. Could there be a complete failure of translation or explicability between two languages? If so, these languages could give rise to philosophical problems that cannot be translated or explained. The problems would therefore not be shareable. Moreover, such completely untranslatable and inexplicable languages would amount to two different conceptual schemes in a strong sense. And against this idea, there are arguments by Davidson and Wittgenstein to the effect that anything that could count as evidence that a system of sounds is a language would thereby help to translate that language (Davidson 1973; *PI*, § 206–7).

But there may be still obstacles in the translation or explanation of philosophical

[5] A similar point is made by Hanfling, who claims that our knowledge of the use of words is *participatory* instead of empirical. He supposes that this is supported by the fact that language is a participatory activity: "[i]f I use words in abnormal ways, I shall be under pressure to normalize my usage. I shall fail to get what I want form others." (Hanfling 2000, p. 54). I agree that the participatory aspect of language also supports the uniformity of language. But it cannot rule out the possibility and actuality of divergences of use and thus has no grip on the normative rebel objection.

problems across languages. If there is a word-to-word translation available, as in "femme" and "woman" in French respectively English, any formulation of a philosophical problem in one language will be isomorphic to a formulation in the other language. Of course, not all translations are simply word-to-word translations. Glock (2007b, p. 393–398) mentions three kinds of differences between languages that give rise to conceptual diversity, and what we might call conceptual schemes in a weak sense: i) Anisomorphism: there is no word-to-word translation, as in Hindi "chacha" means "paternal uncle", or in evidential languages the verb phrase already distinguishes between hearsay and direct observation. ii) Untranslatability: a translation requires enriching the language into which one translates, as a translation of modern mathematics or science into ancient Greek. iii) Inexplicability: a translation is completely hopeless for contingent reasons, as when we have no access to samples due to the way our cognitive apparatus is built. If intelligent aliens would see in the ultra-violet range, and had a term for this, we could not use a synonymous expression, since we are unable to recognise instances of that colour. Though like the blind, we might learn a related but different and dependent expression: "Please pass me the trousers of the chrunnz colour." (This of course presupposes that our colour terms are not synonymous with statements of wavelength. Thus, the existing English word "ultraviolet" wouldn't be a faithful translation.)

How exactly a difference in conceptual schemes so understood and a difference in those philosophical problems that can be formulated in a language hang together is a complex question, which I am unable to tackle here in depth. Let me just make two observations. First, there are cases of anisomorphism that have no bearing on some philosophical problems. For instance, if the colour space is divided up differently, say in only three colours, the concepts involved are clearly different. But assuming that these concepts work in the same way as ours, as determinates of a determinable that mutually exclude each other, etc., the philosophical problems that we have with colours can be formulated in that different conceptual scheme. One such common problem of these different conceptual schemes would be how to understand "being darker than", especially how to grasp the difference between a body being darker than another body and a colour shade being darker than another colour shade (*ROC*, § 1). Even if one wants to take the extremist line that they have not exactly the same problem *because* they employ different concepts, one has to admit that the problems they face are strikingly similar and that they can learn from each other. I prefer to say that they have different concepts but the same problem, but nothing important turns on that.

The second observation is that if a language lacks certain concepts, then there is no way to formulate problems regarding that concept. One example is the concept of *infinity*, which gives rise to much philosophical discussion. Thus, for a tribe that counts only to five and which calls all collections with more than five elements "a lot" (Pica et al. 2004, p. 500, Gordon 2004, p. 496), problems concerning the concept of infinity cannot crop up. But perhaps one could object here that insofar it is possible for that tribe to acquire modern mathematics, the philosophical problems associated with it can be made intelligible for these people. Thus, the problems of infinity are potential problems in a secondary sense: by enriching their language, the problems could be formulated in it. Thus, even if the problem cannot be translated, it can be made intelligible. These two observations point in the same direction. The first states that in cases of anisomorphism, the same problem may still occur in two languages. The second states that while in cases of "intranslatability" (lacking concepts), the problem may be stated in one language but not in another, but insists that the problem is still in principle accessible for human beings. This makes the view plausible that philosophical problems are not mutually inaccessible for human beings that speak languages that amount to different conceptual schemes.

Another reason for convergence of philosophical problems are anthropological facts about common needs, goals, desires, etc. In the words of a title of a film of Ang Lee: "Eat Drink Man Woman". The common human way of doing things serves as the framework for interpreting foreign languages (*PI*, § 206). Moreover, our forms of language are deeply rooted in us (*PI*, § 111) – we couldn't simply abolish a concept like *pain* by legislation, we would not get very far: we may ban a sequence of signs, p-a-i-n, but a different sign with the same role in our language would soon emerge (cf. Hanfling 1989, p. 145). While human nature does not directly dictate that certain concepts must occur in every human language, it certainly demarcates certain topics, preoccupations or themes as very important to us. Assuming that the evidential language mentioned above lacks the concept of *knowledge*, its evidential grammar would be an alternative way to deal with evidence and certainty. Consequently, translatability of conceptual schemes and shareability of philosophical problems are guaranteed to a greater degree among human beings than in an interstellar exchange. An alien civilisation may not share our preoccupations. For all these reasons, investigating the actual use of words does not lead to philosophical isolation.

Concluding Remarks

I end with a short example of how considerations of actual use may come into play in a philosophical investigation. Let's suppose that a graduate student claims that there is such a thing as "false knowledge".[6] That is, if my interlocutor denies the standard explanation of the meaning of the expression *to know*, according to which knowledge claims imply truth claims, I first have to acknowledge this disagreement. Then I have to check whether this testimony is correct – does he actually use *to know* differently than I and the great majority of English-speakers do? If the answer is yes, we have noted a difference of concepts and meaning. Then, obviously, the standard explanations of that word will have no application for *his* concept of *knowledge*. That, of course, does in no way invalidate my explanations of the term "to know" – the basis for doing that would be *my* use. But such an outcome is by itself very interesting, for we have two alternative ways of describing the world at our hand. As Austin writes: "A disagreement as to what we should say is not to be shied off, but to be pounced upon: for the explanation of it can hardly fail to be illuminating" (Austin 1961a, p. 184). If the answer is no, it turns out that my interlocutor has given a false explanation of the meaning of *to know*. That is, I, and perhaps he too, now know that his description is wrong. But, if there is a line of reasoning that lead him to the false description in the first place, we have a philosophical problem at hand, whose treatment depends on its sources and the conceptual apparatus that my interlocutor employs.

[6] Catrin Misselhorn claimed in a discussion that 50% of her undergraduate students thought that there was "falsche Erkenntnis".

II

History and Exegesis

3

Frege, Vagueness, and the Demand for Sharpness

Frege is, besides Peirce (cf. Williamson 1994, ch. 2.3, Peirce 1905), the first analytic philosopher who had views on vagueness. Moreover, the metaphor of unsharp boundaries, which seems to be closely connected to vagueness, has its origins in Frege's writings. In Frege's philosophy, we encounter sharpness as a demand: a concept which can be recognised by logic must be sharp.

Fregean Indeterminacy: A concept F is unsharp iff, for at least one object a, it is neither determinate that a is F nor determinate that a is not F.

Frege famously distinguished between *Sinn* and *Bedeutung*, where *Sinn* can be roughly explained as the meaning of an expression, and *Bedeutung* as its reference or its contribution to the truth-value of the whole sentence. Logic is concerned only with truth, and therefore only with *Bedeutung*. But for Frege, sense has an important role, too. Sentences express thoughts (their sense) independently of having a truth-value. And it is the thoughts which thinkers grasp and judge, and which lead them to their truth-values. Therefore, the laws of logic, which pertain to the realm of *Bedeutung*, are at least indirectly concerned with sense (*ASB*, p. 31-34). The *Bedeutung* of a proper name is the object it refers to; the *Bedeutung* of a concept-word is a concept. A concept is a function which maps arguments onto truth-values. For instance, the concept denoted by "lies in France" maps Paris to "true", and Berlin to "false". According to the Law of Excluded Middle, every proposition is either true or false. For Frege, this law is at root the same as the demand for the sharpness of concepts: If a concept is not sharp, then it is not determined for some object a whether it falls under the under the concept or not (*GLA*, p. 168; *GGA*, Vol. II, § 56). Consequently, there is a proposition which is neither true nor

false. Based on this motivation Frege excludes unsharp concepts from logic, and even claims that they have no *Bedeutung* at all. That is, a proposition which contains an unsharp concept never has a truth-value.

Given that vagueness is ubiquitous in natural language, this leads to the disastrous consequence that most propositions of natural language have no truth-value. This has lead some Frege scholars to mount a rescue attempt, while Kemp (1996) argues that Frege cannot escape nihlism. Burge (1990) proposes a version of epistemicism as an interpretation of Frege: The concepts of natural language are sharp, and we have so far no fully grasped the concept. A more recent attempt is Puryear's (2013): He interprets Frege such that propositions of natural language do have a truth-value, except in those cases in which nothing is determined. This would dampen the nihilistic consequences. That is, for the purposes of ordinary life, but not for the purposes of logic, unsharp concepts do have a *Bedeutung*. However, before we can assess Frege's views on vagueness, we have to get clear on the relation between modern vagueness and Fregean indeterminacy. In section (3.1), I elucidate modern vagueness, and in section (3.2) what Frege means by "sharpness". I then explore the nihilism of the standard interpretation (section 3.3), and consider Burge's and Puryear's alternatives (section 3.4).

3.1. Current uses of "vagueness"

In the modern discussion, the most important word is "vagueness", while the most important problem to be solved is the sorites paradox. While there is no generally accepted definition, there are two explanations and one feature that recur in the discussion. The first explanation is the link between vagueness and the sorites paradox. Indeed, the Stanford entry on the "sorites paradox" draws a close connection between the two: "[The] phenomenon at the heart of the paradox is now recognised as the phenomenon of vagueness." (Hyde 2005) Moreover, the two have been closely tied to the notion of unsharp or blurred boundaries. Williamson (1994, p. 36) writes: "The word ["vagueness"] has been appropriated as a term of art for the phenomenon of blurred boundaries, which results in sorites susceptibility." Examples where vagueness is elucidated and explained in terms of unsharp boundaries *and* the sorites paradox abound (cf.Keefe 2000, p. 6; Smith 2008, p. 10). This establishes that in the modern discussion, *vagueness* is explained by reference to the sorites paradox, and that the notion of *unsharp boundaries* is closely connected to those two notions.

The second way to explain *vagueness* is by appeal to the Tolerance principle and to examples of a certain sort which involve a gradual transition (cf. Wright 1975). A predicate F is tolerant if small changes in an underlying dimension φ (such as the number of grains) do not warrant a change in the correct applicability of F, whereas large changes in φ do. For instance, the famous painter Rembrandt was young at some stage, say with seventeen. Unavoidably, as the seconds pass, he gets older. Now, since *young* is tolerant, the passing of a second never warrants Rembrandt's suddenly becoming old. But by no stretch of the imagination is a sixty year old Rembrandt young. So again, here we have a paradox, which is at root nothing else than the sorites paradox.

Thirdly, an important feature of modern *vagueness* is higher-order vagueness. Not only is it unclear when Rembrandt became *not-young*, but the borders of the "intermediary area" are blurred, too. So it is no solution to the paradox to distinguish three areas in the application of a predicate: the clear cases of F, the clear cases of not-F, and the unclear cases. For it is itself unclear, where the unclear cases begin.

The picture of ordering people according to their age in seconds and then splitting them into *young* and not *young* is highly suggestive. The term "unsharp boundary" can, in this context, be easily understood as follows: The clear cases of *young* are separated from the clear cases of *not-young* not by a sharp, extensionless line, but by a blurred line, which covers some of the objects, but for which there are also intermediate cases (so to speak second-order intermediate cases), for which it is unclear whether they are covered by the blurred line or not. As if hundreds of recruits are told to stand in a single line, which an aeroplane crosses at a right angle and sprays colour at a low level: Some recruits will be fully drenched in colour, some half-way, and many will have some sprinkles on their uniform.

Thus, the term "unsharp boundaries" has a narrow sense in the modern discussion: It is closely tied to the sorites paradox and to gradual transitions. Concepts with unsharp boundaries are vague, and vague concepts are susceptible to the sorites paradox. This sort of vagueness is also called degree or one-dimensional vagueness, and can be contrasted with combinatory or multi-dimensional vagueness (see section 6.2)

Modern Vagueness: A concept F is vague in the modern sense iff, concerning the transition zone, it is not determinate whether an object a is F, nor determinate that object a is not F.

This definition is not neutral concerning the main theories of vagueness: Epistemicism and Degree Theory have different accounts of what vagueness is, while it is more favourable for Supervaluationism or Three-Valued Logic. Nevertheless, it is useful since it allows to see clearly what connection one can draw between Fregean indeterminacy or unsharpness and modern vagueness: The latter is a subclass of the former. Vagueness is indeterminacy that occurs in the transition zone. Or in other words: The exceptions to the Law of Excluded Middle lie in the transition zone.

In the following, I will use the term "vague" for the modern, narrow phenomenon of vagueness, and the terms "unsharp boundary" and "unsharp concepts" in Frege's wide sense: indeterminacy. The term "unsharp concept" is for Frege, of course, a *contradictio in adjecto*. Nevertheless, for the sake of shortness, I will use that locution in the following. Frege would prefer to talk about "Begriffsähnliche Bildungen" (concept-like constructions) (*GGA*, § 58), or about concept-words that do not refer to a sharp concept.

3.2. Frege's understanding of "concept with unsharp boundaries"

This modern understanding of "unsharp boundaries" is not the same as Frege's. What Frege understood by "concept with unsharp boundary" can be seen from his definitions and from his examples. Consider the earliest definition in (*GLA*, § 74):

> Alles was von Seiten der Logik und für die Strenge der Beweisführung von einem Begriffe verlangt werden kann, ist seine scharfe Begrenzung, daß für jeden Gegenstand bestimmt sei, ob er unter ihn falle oder nicht.

Formulations that differ only in the wording can be found all over Frege's work (for instance in *NS*, p. 168; 194, *FB*, p. 135). Frege is very clear that he demands from any concept that it be sharply bounded, and explains this as the demand that for every object, it must be determinate whether it falls under the concept or not. Clearly, a Fregean unsharp concept is not *per se* the same as a vague concept in the modern sense: the indeterminacy is not restricted to the boundary at the transition. Nevertheless, there is a connection: For vague concepts are a subclass of indeterminate concepts. Indeterminacy, on Frege's definition, does also include terms that are incompletely defined such as *nice number* (a number is nice if $n > 17$ and not-nice if $n < 13$ and nothing else is specified) (Fine 1975) or *happy number* (a number is happy if $n < 15$ and not happy if $n > 15$ and $n < 23$ and nothing else is specified).

While the demand for sharpness is formulated for concepts, it has consequences for any sort of function. In (*FB*, p. 131), Frege extends the notion of function in order to make it usable for logic: "=" and "<" are new mathematical operations which can create a function; and new objects can serve as arguments and function-values. Functions which have truth-values (and nothing else) as values are concepts. Now, since concepts can be constructed from non-conceptual functions, and the thus-constructed concepts have a *Bedeutung* only if all its logical parts have a *Bedeutung*, the demand for the sharpness of concepts generalises into the demand that all functions have a value for each argument. This is a consequence that Frege draws himself (*FB*, p. 135, see also Kemp 1996, p. 172). Take the non-conceptual function

$$x + 1$$

For the argument 2, the function's value is 3, for the argument 3, its value is 4. A concept can be constructed with that function as a part, for instance:[1]

$$x + 1 = 10$$

For the argument 2, the value is "the false", for the argument 9, the value is "the true". (In contrast, the sign "$2+1 = 10$" expresses a sentence, which is false.) Since the function "$x + 1$" is a logical part of the concept "$x + 1 = 10$", that the concept fulfils the demand for sharpness depends on the function "$x + 1$" having a value for each argument. Now, mathematical functions like "$x + 1$" had been defined for mathematical objects only. But since Frege extended what can be an argument of a function, it must now also have a value for non-mathematical arguments like the material object denoted by "the sun". He is explicit in that this will be a by and large arbitrary stipulation:

> Wie diese Festsetzungen geschehen, ist verhältnismäßig gleichgültig; wesentlich ist aber, daß sie gemacht werden." (*FB*, p. 135)

So we could just stipulate that the addition "the sun +1" produces 5, or we could say it produces the Venus, or 10'000 chicks, or whatever. Frege also recognises linguistic functions like "the capital of x" (*FB*, p. 134). For the argument Germany, it it's value

[1] That a mathematical equation is a concept is not something that one would say in ordinary English. However, it follows from Frege's definition of a concept, and it is witness to Frege's mathematisation of logic.

is Berlin, for the argument France, its value is Paris. Given the reasoning above, this function must also have a value for the arguments 5, Roger Federer, Mount Everest, Paris, or the current President of the EU-commission. The generalisation of the demand for sharpness of concepts to a demand for the completeness of functions makes clear that Frege was not concerned with lack of determination at a transition zone, but with a lack of determination at any place of the value-range of functions.

The point can be strengthened by examining the examples of unsharp concepts which Frege considers. There are three I know of. I will present them in chronological order (apart from the *acervus*, which I will treat last). The first and most interesting example is in the essay *Über das Trägheitsgesetz* (*KS*, p. 113, 122–123), where he speaks of the "incompleteness of the expression [...] "a moves"," linked to the question whether a man who walks backwards in a moving ship moves or not. This indeterminacy leads to different people giving different answers, that is, to contradictions. The remedy in that case is quite simple: Acknowledge the incompleteness of the notion and replace it by "a moves in relation to b". This amounts to a sharpening of the concept: An absolute conception of *movement* has been replaced by a relative one. Frege makes clear that he does not hold that the *concept* is contradictory, but that the contradictions arise from the *indeterminacy* of the concept:

> Auch in unserm Falle sind es keine Widersprüche in dem Begriffe der *Bewegung*, die zur Entwickelung treiben. Allerdings haben sich Widersprüche gezeigt, aber nicht so, daß man etwa einander widersprechende Merkmale in der Definition vereinigt hätte, sondern dadurch, daß man etwas als Begriff behandelt hat, was im logischen Sinne keiner ist, da die *scharfe Begrenzung* fehlt. (my emphasis, *KS*, p. 123)

A second example is the concept *Christian* in (*GGA*, Vol. II, § 56)[2]. Frege does not elucidate the example, so there is no way of knowing the exact reason why he considers this an unsharp concept. This example occurs in the context of the famous passage where Frege compares a concept to an area, to which Wittgenstein refers in (*PI*, § 71):

[2]Interestingly, in that passage, Frege does not deny that "We are still Christians" has a truth-value. Rather, he goes further and questions whether the proposition has *sense* at all. That would be meaning-nihilism, not truth-nihilism. Nevertheless, I stick to ascribing truth-nihilism to Frege. For one thing, as far as I know, this is a solitary remark. For another, Frege does not deny outright that the proposition has a sense, but rather asks whether it has a sense.

> Wenn man sich Begriffe ihrem Umfange nach durch Bezirke in der Ebene versinnlicht, so ist das freilich ein Gleichnis, das nur mit Vorsicht gebraucht werden darf, hier aber gute Dienste leisten kann. Einem unscharf begrenzten Begriffe würde ein Bezirk entsprechen, der nicht überall eine scharfe Grenzlinie hätte, sondern stellenweise ganz verschwimmend in die Umgebung überginge. (*GGA*, Vol. II, § 56)

To my knowledge, this passage is the origin of the metaphor of unsharp boundaries which plays a central role in the modern discussion of vagueness. One difference to the modern discussion is that the unsharp boundary is not imagined as dividing a one-dimensional line, but as demarcating an area on a two-dimensional plane. This might suggest a similarity to the modern idea of multi-dimensional vagueness, where more than one dimension is relevant for the applicability of a predicate (Alston 1964, p. 87, see also Keefe 2000, p. 11). Moreover, the example does not specify whether the objects on the plane are ordered according to similarity. So this could be read either way. However, it would fit Frege's definition of sharpness that they are not imagined as being ordered. Accordingly, a blurred boundary of an area corresponds to a lack of determinacy: For some objects, it is not determined whether they fall under a concept F or whether they do not (whether or not they strongly resemble a determinate case). To the modern reader, this understanding of *unsharp boundaries* may look like a misleading use of the metaphor; but then again, this has to do with our own habits of understanding it.

The third example is of special interest because it coincides with Frege's treatment of the sorites paradox, which he discusses under its Latin name, "acervus". It is shortly mentioned in (*BS*, § 27), where the concept *heap* is described as indeterminate. His clearest (if still concise) treatment of it can be found in the essay *Begründung meiner strengeren Grundsätze des Definierens* (*NS*, p. 168), which he later repeated in the famous letter to Peano (*BW*, p.183). Frege's treatment of the acervus is simply a dismissal of it as a fallacy: *Heap* is not sharply bounded, and can thus not be treated as a concept by logic. In other words: Logic, and with that, logical inferences, do not apply such defective formations. Frege's position on the sorites paradox is prone to misunderstandings if one does not carefully distinguish between modern vagueness and Fregean indeterminacy. Williamson is a point in case:

> Frege sometimes speaks of sorites paradoxes as the result of treating a vague predicate as though it stood for a sharp concept. His description is not quite

> right, for someone who does think that "heap" stands for a concept with a sharp cut-off point should not agree that one bean never makes the difference between a heap and a non-heap. (Williamson 1994, p. 43)

While Williamson's description of Frege's strategy is correct, his critique of Frege is beside the point. For Williamson, somebody who thinks that *heap* is a sharp concept (and applies logic to it) would think that the Tolerance principle is false.

Tolerance Principle: If *n* grains make a heap, then *n–1* grains make a heap, too.

But if he does so, then there is a sharp cut-off point (a number of grains n, for which holds: n grains make a heap, but n-1 grains do not), and no reason to succumb to the fallacy of the sorites paradox. Thus understood, in Frege's framework, the sorites paradox would be a trap into which nobody would ever fall. The mistake is, of course, that this ascribes a modern understanding of sharpness to Frege. If we take Frege's understanding of sharpness, then his description of why we fall for the sorites paradox makes more sense. If *heap* were a sharp concept in Frege's sense, this would be compatible not just with rejecting Tolerance, but also with accepting it. The reason is that in both cases, it is determinate for every object whether it falls under the concept of heap. However, one might argue that if Tolerance is true, a contradiction can be developed, and that *heap* is a contradictory concept. But is a contradictory concept a concept at all, one then may ask. However, for Frege, the answer to that question is clear: Contradictory concepts are indeed concepts; their extension is empty. A contradictory concept even plays a crucial role in Frege's definition of *number* (*GLA*, p. 87).

Of course, for Frege, *heap* is an unsharp concept. And in that case, neither Tolerance nor its negation (for "heap") are true, because "heap" is not truth-apt in the first place. The unsharp concept *heap* defies the Law of Excluded Middle. Accordingly, Frege can say that the source of the sorites paradox is to treat the unsharp concept as a sharp one. Crucially, to accept Tolerance is to regard the concept *heap* as a sharp one. And this idea leads to the sorites paradox. Frege's solution to the paradox consist in pulling the rug from underneath the assumption that one can reason with unsharp concepts. Logic does not apply to unsharp concepts, and no inference is possible using such concepts. Indeed, neither Tolerance nor its negation are true for unsharp concepts, they are not *bona fide* concepts to begin with. Frege's solution is straightforward, but so radical that it has its own disadvantages. As we will see, it is committed to nihilism.

In none of the examples, Frege mentions anything that resembles higher-order vagueness, nor is there any allusion to a gradual transition. Even in the *BS*, where *heap* is treated in the context of mathematical series that inherit properties (which may look like a gradual transition, but is not, since there is no transition), he only speaks of certain objects, for which *heap* does not yield a judgeable content. Nevertheless, those are the examples that he links to unsharp boundaries. Together with his definition, this should make it crystal clear that Frege was not at all concerned with what modern analytic philosophers call "vagueness".

The linguistic confusion about "unsharp boundary" engenders two complementary inaccuracies. On the one hand, there are the Frege scholars who know quite well that Frege is concerned with indeterminate concepts (of which vague concepts, in the modern sense, are a subclass), but nevertheless ascribe to Frege views about "vagueness", without mentioning that Frege never used that term, and thus projecting the modern discussion of vagueness onto Frege (Van Heijenoort 1986, Ruffino 2003, Puryear 2013).[3] Puryear (2013, p. 120–121) for instance gives a roughly Fregean definition of vagueness, only to later refer to Williamson's position on vagueness, thus conflating the two distinct discussions. On the other hand, there are experts for vagueness who find Frege guilty of mixing together two distinct things: vagueness and incompleteness of definition (Dummett 1981, p. 646, Williamson 1994, p. 38). But Frege cannot be accused of such an act, since he simply was not concerned with modern vagueness, but only with what he calls "unsharp boundaries", incompleteness of definition. His mentioning the acervus is not a case of dealing with vagueness, but only of assimilating the acervus to his incompleteness scheme. However, what is right is that Frege's views on determinacy have implications on the issue of vagueness in natural language.

3.3. Frege's truth-nihilism

Vague concepts in the modern sense are a subclass of unsharp concepts in Frege's sense. Therefore, what Frege says about unsharp concepts is true for vague concepts, too: They lack a *Bedeutung* (*NS*, p. 133). But then, sentences that contain such concept-words are incomplete, at least in terms of truth. For instance, in "Albert Einstein is bald" the

[3] I have not found "Vagheit" or "vage" in *GLA*, *BS*, and *GGA*. In *KS*, there is a letter *from* Hilbert to Frege (*KS*, p. 411). However, no searchable copies of the two volumes of *NS* are available to me. The two occurrences of "vague" that Ruffino mentions are translations of the German words "dehnbar" and "unbestimmt".

proper name "Albert Einstein" refers to an object, but the concept-word "is bald" does not refer to a concept, thus, in terms of truth, no mapping of the object *Albert Einstein* to a truth-value takes place, and the sentence lacks a truth-value. This consequence is systematic: Every sentence that contains an unsharp concept has no truth-value.

Note however that Frege does not claim that the sentences in question have no *Sinn* (sense), and Frege's technical notion of *Sinn* is that which comes closest to the ordinary distinction between meaningful and nonsensical sentences. It is important to keep in mind that, while the German word "Bedeutung" should normally be translated as "meaning", the way that Frege uses "Bedeutung" has little to do with its ordinary meaning. That is why the standard translation is "reference" (Geach/Black 1977, p. ix). Moreover, for a proposition to have a meaning is to have a truth-value, and the *Bedeutungen* of its parts are relevant for the *Bedeutung* of the proposition (*Über Sinn und Bedeutung*, p. 34, 49). Thus, for Frege, if a part of a proposition has no referent, then the proposition has no truth-value. Sentences that have a *Sinn* but no *Bedeutung* are "*Schein-Gedanken*", mock-thoughts, they belong to the realm of fiction (*NS*, p. 141). The primary case for fiction are proper names that have no referent like "Skylla", "Wilhelm Tell" or "Gregor Samsa". Yet it is not surprising that empty proper names and unsharp concepts receive an analogous treatment. Logic demands only one thing from concept-words and proper names: Proper names should have exactly one referent and concept-words should be sharply bounded. If they don't fulfil that requirement, they don't refer to an object or a function, and propositions that have them as parts are thus not truth-apt, but still express a sense.

Of course, this treatment is much more plausible in the case of proper names. A missing referent is indeed very often a case of fiction (disregarding definite descriptions), and fiction has indeed not the same claim to truth as other assertions. Nevertheless, for ordinary language, some aspect of truth can be claimed even for discourse about fiction. If someone claimed that the cookie monster doesn't like cookies, an appropriate reply would be to say: "That's false! Of course the cookie monster likes cookies. He loves them!" Be that as it may, for unsharp concepts in ordinary language the claim is much more implausible. Most if not all of our concepts are vague in the modern, narrow sense, and this implies that for the borderline cases, it is not unquestionably determined whether they fall under the concept in question or not. But then most if not all ordinary language sentences turn out to be mere fiction, without any truth-value!

Such an outcome can rightly be called nihilism, or more exactly truth-nihilism, because

practically no sentence of ordinary parlance will turn out to be true, such sentences will be devoid of any truth-value. The realm of truth, according to Frege's nihilism, is restricted to those areas where his *Begriffsschrift* applies, namely logic, mathematics and sciences like mechanics and physics (*BS*, p. XII). This truth-nihilism concerning unsharp concepts differs from the existence-nihilism (concerning degree vague concepts) of Unger (1979) and Heller (1990) which claim that sentences like "There are bananas" are false, and that such things as heaps and bananas do not exist. What Frege's truth-nihilism amounts to in terms of existence, is an open question. Both statements "There are φ" and "There are no φ" do not have a truth-value.

Weiner (1997, p. 264) tries to defend Frege's truth-nihilism by claiming that for the purposes of communication, truth doesn't matter. The speaker's description "the bald man in the corner" might enable the hearer to identify the relevant man even if the description is false, namely in case the man wears a skull cap which hides his hair. But this is misguided for a couple of reasons. First, Frege's truth-nihilism commits him not to communication-despite-falsity but to communication-despite-truth-value-gap. Second, Weiner argues with a special case: Both the speaker and the hearer are wrong in their judgement, but wrong in the same way. While it's true that in this case, insisting on the truth would perhaps make things more complicated, in general, a false description leads to a failure of identification. If I say: "Please pick up Martin from the station. He will arrive with the three o'clock train, wears a red sweater and is bald," and what I say is wrong (he might arrive with a different train, or wear a blue sweater, or sport a trendy hair-cut), my hearer will probably not be able to identify the man and thus my communicative purposes will not be satisfied. Finally, unsharp concepts are not exempt from truth-talk. "False! His sweater is not red, but violet." For these reasons, truth-nihilism concerning ordinary talk is completely misguided.

In addition to truth-nihilism, Frege is committed to a different kind of nihilism about unsharp concepts: inference-nihilism.[4] The core business of logic is to provide a guide to

[4]Glock (2013, p. 254-5) argues that Frege is committed to another implausible position regarding inference: That one can draw inferences only from true premises. See (Künne 2009) for a dissenting view. For my part, I don't think that Frege's view is obviously implausible. By distinguishing hypothetical reasoning from drawing inferences, he can accommodate the fact that propositions have logical relations independent of their truth. However, the hypothetical reasoning strategy does not work for the *reductio ad absurdum* argument form. There, an asserted proposition is inferred from a mere assumption, and the assumption is not part of the conclusion. We could not say "If p were the case, q would be the case", but should say "If p were the case, not-p would be the case". But this is not the point of a *reductio*.

separate valid arguments from invalid arguments. But since logic does not apply to huge swathes of ordinary talk, it can no longer fulfil that role for ordinary language – most of our ordinary reasoning, often only tacitly present, comes out as invalid. "There is nothing to eat left in our house." – "We have some cookies, and therefore something to eat." Or "Your sister wore a beautiful blue dress at the dinner party yesterday." – "That's false. I've been with her the whole evening and she wore a yellow one throughout."

3.4. Alternative interpretations of Frege

There is some evidence that Frege was aware of the problem of nihilism, since he makes some gestures towards a solution in the letter to Peano:

> Der unter dem Namen "Acervus" bekannte Trugschluß beruht darauf, daß Worte wie "Haufe" so behandelt werden, als bezeichneten sie einen scharf begrenzten Begriff, während dies doch nicht der Fall ist. [...] so muß die Logik scharfe Begrenzung von dem verlangen, was sie als Begriff anerkennen kann, wenn sie nicht auf Genauigkeit und Sicherheit verzichten will. Ein Begriffszeichen also, dessen Inhalt dieser Forderung nicht genügt, ist vom logischen Standpunkte aus als bedeutungslos anzusehen. Man kann einwenden, daß solche Wörter in der Sprache des Lebens tausendfach gebraucht werden. Ja! aber unsere Volkssprachen sind auch nicht dazu geschaffen, Beweise zu führen. Und ihre hieraus entspringenden Mängel sind grade der Hauptgrund für mich gewesen, eine Begriffsschrift aufzustellen. Die Aufgabe unserer Volkssprachen ist wesentlich erfüllt, wenn die miteinander verkehrenden Menschen mit demselben Satze denselben Gedanken verbinden, oder doch annähernd denselben. [i] Es ist dazu nicht durchaus nötig, daß die einzelnen Wörter für sich einen Sinn und eine Bedeutung haben, wenn nur der ganze Satz einen Sinn hat. Anders liegt die Sache, wenn Schlüsse gezogen werden sollen; denn dabei ist wesentlich, daß in zwei Sätzen derselbe Ausdruck vorkomme, und daß dieser in beiden genau dieselbe Bedeutung habe. [ii] Er muß also für sich eine Bedeutung haben, die unabhängig ist von den andern Teilen des Satzes. (Letter to Peano, 29.9.1896, *BW*, p.183)

Reinterpretations of Frege, which would allow him to eschew the charge of nihilism, use this as a core passage. And indeed, here we have an explicit mention of an unsharply

bounded 'concept', namely "heap", and an allusion to the sorites paradox. Moreover, the task of natural languages is contrasted with the task of logic: The task of natural language is to enable the "Verkehr" (dealings) between human beings. The task of logic is to conduct rigorous proofs and draw inferences. Thus, Frege remains committed to an inference-nihilism, even if defended on the basis of this passage. Inference-nihilism claims that it is very imprecise and insecure to draw inferences with words of natural language and that logic applies only to concepts that meet the demand for sharp boundaries.

I begin with an interpretation that is mentioned but does not receive much credence in the literature. It relates to the sentence in the quotation that I labelled with (i). According to it, vague sentences are treated as *idioms*: Their sense does not depend on the sense of their parts. An example of an idiom is "to pay the piper". It means to face the consequences of one's actions but is not restricted to payments to players of the bagpipe, as the words of idiom suggest. If vague sentences are treated as idioms, they have no logical parts which could fail to have a reference, and the failure of reference of unsharp concepts would not pose a problem. However, as Williamson (1994, p. 44) is quick to point out, new sentences using unsharply bounded concepts can be understood without learning the meaning of the sentence anew: "When he opened his bag, an avalanche of sand spilled out to form a heap."

A more promising proposal is the one by Burge (1990, see also Van Heijenoort 1986, p. 37), who tries to limit the scope of nihilism by claiming that many or most words *are* in fact sharp for Frege. The way to achieve this is to embrace a form of *epistemicism*: the *sense* of certain words is only incompletely grasped by human thinkers, and if they fully grasped the sense, the concept would turn out to be sharp. Moreover, he claims that for Frege, sense transcends linguistic practice: "No amount of investigation of the actual usage and understanding of one's language is, according to Frege, sure to reveal the 'deeper structure' and nature of sense and thought." (Burge 1990, p. 32) Because *Bedeutung* is mediated by sense, an unclear grasp of sense leads to an unclear grasp of *Bedeutung*. Unclarities concerning whether a particular object falls under a concept are, according to Burge's epistemicism, a consequence of the incomplete understanding of the sense of a word. Burge plays down the extent into which the remarks from the letter to Peano can be generalised and takes the cue from Frege's discussion of Weierstrass, to whom Frege ascribes an unclear grasp of the notion of number (Burge 1990, p. 35, 39, *NS*, p. 239). This supports the view that at least for mathematical concepts, and perhaps for those of the natural sciences, the problem is not that the concept is unsharp. Rather,

it is that the scientific community has yet only an unclear grasp of those concepts. So there is exegetical evidence that scientific and mathematical statements are exempt from truth-nihilism. However, Burge tries to apply this epistemic model to ordinary discourse, too:

> If [Frege] were to hold that nearly all concept words in mathematics, natural science, and ordinary discourse are vague, and thus lack a *Bedeutung*, he would have to hold that nearly all sentences in actual use strictly speaking express neither truths nor falsehoods. There is no suggestion of a 'secret doctrine' to this effect. (Burge 1990, p. 37)

What Burge overlooks is that Frege is not putting forward a "secret doctrine", but that what Frege says about unsharp concepts has consequences for ordinary discourse. Frege does not propound truth-nihilism for ordinary discourse, but he is *committed* to it. The most charitable reading of Frege, and here I concur with (Kemp 1996, p. 173), is that Frege missed these disastrous consequences. What supports this view is that he may have regarded the word "heap" as a special case. I will not discuss the merits of Burge's interpretation for scientific or mathematical concepts. Important for us is that this strategy fails as a defence against the charge of (truth-) nihilism for ordinary discourse. For one thing, the plausibility of Burge's model is limited to mathematics. For instance, it is a long-standing view that when geometricians study the properties of circles, they do not study the properties of a particular circle drawn on paper, whose lines have thickness, but those of an abstract object. But what abstracts objects are to be studied in order to grasp the hidden *Bedeutung* of "underpants", "Samosa" or "meal worm"? Thus, Burge's particular model of epistemicism which incorporates practice-transcendent abstract objects has little plausibility for ordinary concepts (cf. Kemp 1996, p. 181).[5] For another, Burge underestimates the extent of the threat of truth-nihilism when he writes: "When [Frege] discusses vague, denotationless concept words, the examples he offers are always presented as if they were special cases – 'heap', 'bald' and so on." (Burge 1990, p. 37).[6] The problem is that Frege's remarks on "heap" in the letter to Peano generalise. For "heap" is connected to the sorites paradox. But then, any notion which is susceptible

[5] A more plausible line of thought would be that natural (and artificial) kinds determine the alleged sharp boundaries of these terms. See (Putnam 1975) for a classical statement of the causal theory of meaning, and (Hanfling 2000, ch. 12) for objections.

[6] I have not been able to find the term "bald" and its German cognates at the passages indicated by Burge. Therefore, it is not a fourth example of an unsharp concept that Frege considers (see p. 47).

to the sorites paradox is also an unsharp concept (Puryear 2013, p. 126). In other words, huge swathes of ordinary talk are not truth-apt.

Another attempt to save Frege from nihilism is Puryear's (2013, p. 127–37). His interpretation is the most elaborated alternative and ascribes to Frege a kind of ersatz theory for the *Bedeutungen* of unsharp concept-words. It combines three ideas: First, the *Bedeutung* of an unsharp concept-word is not a concept, but a concept-like construction. Second, unsharp concept-words lack a *Bedeutung* only in the context of borderline cases, or other undefined cases. Third, he distinguishes between a logical point of view, and one or more non-logical points of view, notably the point of view of ordinary language or ordinary life. An unsharp concept-word U lacks a *Bedeutung* only from the logical point of view. From the point of view of ordinary life, U does have a *Bedeutung*, namely a concept-like construction, but only in those contexts where no borderline cases of U are involved, where U is defined.

For the first point, Puryear (2013, p. 136) reinterprets a passage where Frege talks of unsharp concepts as "begriffsartige Bildungen" or "begriffsähnliche Bildungen" (*GGA*, Vol. II, § 56, 58): According to him, Frege does not claim that unsharp concepts have no *Bedeutung*, but rather that their reference is not a concept, but a "concept-like construction". Thus, there are three kinds of referents of linguistic expressions: objects, concepts and concept-like constructions. While it is true that Frege does not *say* in these paragraphs that unsharp concept-words lack a *Bedeutung*, there is a somewhat curious passage where he insinuates a worse fate.

> Hätte z. B. der Satz "jede Quadratwurzel aus 9 ist ungerade" wohl überhaupt einen fassbaren Sinn, wenn *Quadratwurzel aus 9* nicht ein scharf begrenzter Begriff wäre? Hat die Frage "Sind wir noch Christen" eigentlich einen Sinn, wenn nicht bestimmt ist, von wem das Prädikat *Christ* mit Wahrheit ausgesagt werden kann, und wem es abgesprochen werden muss? (*GGA*, Vol. II, § 56, 58)

Frege ends § 56 with two rhetorical questions which insinuate that the relevant sentences have no sense. But if they have no sense, then they have no *Bedeutung* either, and are not truth-apt. The passage is curious because it seems implausible to claim that these sentences have no sense at all. Granted, I am unable to answer Frege's question "Are we still Christians?", and I am unable because this question, in isolation, lacks clarity. I don't know to what particular aspect of Frege's contemporary situation he

refers as reason to doubt an affirmative answer or who is meant by "we"; and there is no explanation what would count as being a Christian in that discussion. Despite this, I can surmise that some religious aspect of life is relevant. Equally, I do not think that it is obvious that, because there might be square roots of 9 other than 3 and −3 (due to incomplete definition), the statement "All square roots of 9 are odd numbers" lacks a *sense*. Nevertheless, Frege seems to think that there is something terribly wrong with unsharp concept-words. And this makes it implausible to read him as saying that unsharp concept-words have a *Bedeutung* after all, and refer to concept-like constructions.

The second point is that unsharp concept-words acquire a reference only in the right context. The relevant context here is meta-linguistic: If the sentence is about a borderline or other undefined case, the expression does not refer to a *Bedeutung*. If the sentence is about a clear case, it does. So the sentence "This man is bald" has a truth-value if the man referred to is Gorbachev (true) or Bob Marley (false), but not in a borderline case like Wayne Rooney (before the hair transplant). In the latter case, and for the sentence "Wayne Rooney is bald", the predicate "is bald" does not refer to a concept-like construction. It refers to nothing, and the sentence lacks a truth-value.

Puryear (2013, p. 133–4) has two witnesses for this. One is the sentence in the letter to Peano that I labelled with (ii), another a passage from *Begründung meiner strengeren Grundsätze des Definierens* (*NS*, p. 168). Common to both passages is that, in the context of discussing the sorites fallacy and incomplete definitions, Frege claims that for drawing secure inferences, a word needs to have a *Bedeutung* independently of context. In the letter to Peano, this context is very specific. It is the other parts of the sentence. For this can lead to fallacies: "Romeo missed Juliet for a long time and the Tuesday class, therefore he missed two things." In the passage of the *Begründung*, Frege does not specify which context is meant. That means, however, that Frege does not specify the context in the way required for Puryear, namely that incompletely defined words lack a *Bedeutung* only in those cases in which they are not defined. Therefore, I see little explicit support for Puryear's reading.

The third point is the most important one. According to Puryear, Frege does not claim that unsharp concept-words have no *Bedeutung* at all, but that they have no *Bedeutung* from the logical point of view. From the ordinary point of view, as elaborated above, they do have *Bedeutungen*, namely concept-like constructions, though only in cases for which they are defined. It is indeed striking that those passages where unsharp concept-words are denied a *Bedeutung* are often accompanied by reference to a logical point of view

(Puryear 2013, p. 128–9). However, there is a passage that is a potential problem for Puryear's reading.

> Wenn es einem auf die Wahrheit ankommt – und auf die Wahrheit zielt die Logik hin – muss man auch nach den Bedeutungen fragen, muss man Eigennamen verwerfen, welche keinen Gegenstand bezeichnen oder benennen, wiewohl sie einen Sinn haben mögen; muss man Begriffswörter verwerfen, die keine Bedeutung haben. Das sind nicht etwa solche, die Widersprechendes vereinigen – denn ein Begriff kann recht wohl leer sein – sondern solche, bei denen die Umgrenzung verschwommen ist. (*ASB*, p. 32)

Puryear tries to interpret this in the following way:

> [Frege's] point might be not simply that unsharp predicates are *bedeutungslos*, but that they are or must be regarded as *bedeutungslos* from the logical point of view, that is, when we are concerned with truth or logic. (2013, p. 130)

Puryear claims that the conditional in the beginning "if one is concerned with truth" changes the context of the assertion that unsharp concept-words lack a *Bedeutung*. However, this is not entirely convincing. Frege says that if one is concerned with truth, one needs to *ask for the Bedeutungen*. If one is not concerned with truth, as, in Frege's eyes, in fiction, one may contend oneself with *sense* without progressing to *Bedeutung*. This effectively reserves the notion of truth for logic. I agree that Frege condones an ordinary point of view, but this passage suggests that it is no help if one is concerned with truth. Indeed, Frege continues the passage above by considering the concept-word "$\mu\tilde{\omega}\lambda\upsilon$" from Homer's *Odyssey*. For Frege, it is clear that that word lacks a *Bedeutung*, since only some features are given, and it is not clear which plant is meant by "$\mu\tilde{\omega}\lambda\upsilon$".[7] Then, Frege strengthens the analogy to fictional names and claims that Homer's sentences incorporating "$\mu\tilde{\omega}\lambda\upsilon$" need not lack *sense*, even though they lack a *Bedeutung*. But if the ordinary point of view has no access to truth, Frege is still committed to truth-nihilism: Any sentences that includes a vague predicate lacks a truth-value.

Puryear's reading of Frege's ordinary point of view comes close to a truth-value gap approach to indeterminate concepts: They have a truth-value in cases for which they

[7] This only supports the idea that "$\mu\tilde{\omega}\lambda\upsilon$" is indeterminate for us, given the text corpus we have, but not that it was indeterminate in Homer's times.

are defined, and lack one in cases where they are not defined. In particular, statements about borderline cases lack a truth-value. Nevertheless, for the logical point of view, all statements in which indeterminate or unsharp concepts occur lack a truth-value. This is reflected both in how the sorites is treated, and that Puryear's Frege is still committed to inference-nihilism. For Frege, the sorites is a fallacy because an unsharp concept is treated as it were sharp, that is, it is treated as if logic were applicable to it, and inferences could be drawn with it. But it is unsharp, and it has no place in inferences. But then, the ordinary point of view has no access to inferences even if we accept Puryear's reading. In the end, I do not think that Puryear's interpretation is particularly plausible as an interpretation, nor that it affords Frege a view which saves him from all nihilisms that threaten it.

Conclusion

When reconstructing Frege's views on "vagueness", it is absolutely imperative to distinguish the modern sense of *vagueness* and the related modern *unsharpness* from Frege's sharpness requirement for predicates. Vague concepts in the modern sense are susceptible to the sorites paradox and involve a gradual transition. For Frege, however, vague concepts are only a subclass of indeterminate concepts. Any predicate for which it is not determined for at least one object whether this object falls under the predicate or not is unsharp. Accordingly, Frege's requirements for concepts have consequences for vagueness. Frege does not *solve* the sorites paradox, but *dismisses* it.

Nevertheless, these consequences are far-reaching. The requirement of sharp boundaries leads to truth-value gaps for most utterances in ordinary talk. And since Frege's strictures are due to considerations about truth, their consequences for ordinary talk are serious. Thus, Frege's position amounts to truth- and inference-nihilism for vague and other partially indeterminate concepts. The attempts to save Frege from these nihilistic consequences by alternative interpretations are, in my view, unsuccessful. Burge's epistemicism underestimates, as Frege, the ubiquity of unsharp concepts in ordinary language. It therefore cannot insulate Frege's philosophy of language from nihilistic consequences. Puryear's interpretation turns out to be only lightly supported by Frege's actual texts, and even if successful as an interpretation, would not preserve the applicability of inferences to ordinary language.

For Frege, ordinary language is defective in two ways: Not only is its surface grammar

misleading as regards thoughts, but also wide stretches of it are not truth-apt, that is, *logically deficient*. That, of course, should not surprise us, but reflects the inadequacy of an ideal language approach to philosophy. In that respect, the *Tractarian* position constitutes both an advance over Frege's and a natural continuation of it. An advance, because the threat of nihilism is dealt with in one fell swoop, by simply incorporating the perfect order of Frege's artificial language into ordinary language. "All the propositions of our everyday language, just as they stand, are in perfect logical order" (*TLP*, 5.5563). A continuation, because the Fregean demands for sharpness of predicates and non-emptiness of proper names are incorporated into ordinary language. In that sense, the modern proposals of alternative logics for vague terms stands both in the tradition of Frege's ideal language philosophy and the *Tractarian* postulation of such an ideal language hidden beneath the surface of ordinary language.

4

The Demand for Determinacy in the *Tractatus*

The propositions of ordinary language are not vague or indeterminate at all. Moreover, we are not in a position to say what they mean *exactly*. What they mean exactly has to be established through a deep-level analysis. This is the *Tractarian* position indeterminacy. For modern vagueness, this means that the *Tractatus* is committed to a form of epistemicism: The concepts of ordinary language have sharp boundaries, and the speakers are not able to locate these boundaries. This is the interpretation of the *Tractatus* for which I will argue in this paper. It is embedded in the framework of a broadly Hackerian interpretation (Hacker 2001; Schroeder 2006, ch. 2).

The *Tractarian* position is best understood against the backdrop of Frege's views about vagueness, logic and ordinary language. For two reasons: First, Frege, besides Russell, was one of the main influences on the *Tractatus* (*TLP*, pref.), especially its logic (in agreement as well as an object of critique). In particular, the ancestor of the demand for determinacy in the *Tractatus* is Frege's demand for sharpness of concepts. The second reason is terminological. When viewed from a modern viewpoint, early occurrences of "vagueness" or "sharpness" can easily be mistaken to mean the same as they mean in the contemporary discussion: To be about indeterminacy at the transition between objects that fall under a concept and objects that do not fall under it, where the objects are arranged on one dimension according to similarity.

Modern Vagueness: A concept F is vague in the modern sense iff there is a transition zone in which there is at least an object a for which it is not determinate whether it is F, nor determinate that it is not F.

But as I have shown in the last chapter (3), Frege was not concerned with modern vagueness. Neither is the early Wittgenstein. Accordingly, the little what is said about

vagueness, indeterminacy or sharpness in the *Tractatus* or in the *Notebooks* is best understood from a Fregean perspective, where any indeterminacy makes a concept unsharp, and not just an indeterminacy in a transition zone.

Another important terminological issue is to distinguish between indeterminacy (including modern vagueness) and unspecificity.

Unspecificity: A description is unspecific iff it is true in more than one situation.

This of course only covers declarative speech acts. Also, whether a statement (or proposition) can be criticised on the grounds of being unspecific is relative to a standard of specificity. For instance, the utterance "This rod is made of bronze" can be thought to be specific enough, or to be utterly unspecific because it does not specify the mixing ratio of copper and zinc. Here is an unspecific statement:

1) A man who is between one and two metres tall is holding a rod of metal in his hand.

The sentence (1) is unspecific in several respects. First, it doesn't specify who is holding the rod: "a man". Second, the man's height is described, but in very unspecific ways: any height between one and two metres is compatible with the proposition. Third, it is not specified of which kind of metal the rod is made. Thus, proposition (1) is true in many situations and in that sense unspecific. In *Tractarian* terminology, the sentence has a wide truth-range (*TLP*, 4.463). This does not imply that it is indeterminate. As long as it has a truth-value in all situations, it is not indeterminate.

In comparison to Frege, regarding vagueness or indeterminacy, there are two main moves that the *Tractatus* makes. The first is the idea that the propositions of ordinary language are completely determinate even if they don't appear to be. The second main move is that the demand for determinacy together with the picture theory leads to the radical idea of deep-level analysis: For the proposition "the book lies on the table" to be completely determinate, it is not enough that the relation of lying is determinate for every pair of objects. The proposition needs to lay down the permissible configurations of the *simple objects* of which the book and the table are composed.

A major reason for ascribing to Wittgenstein a reformulation of the demand for determinacy is that, in contrast to Frege, he does not recognise concepts as a major logical kind. For Frege, the distinction between objects and concepts is absolutely

fundamental. They are the two major logical kinds (with quantifiers as second-order concepts perhaps a third, minor one). Moreover, two important demands of logic are formulated for proper names and for concept-words. Proper names should have a reference, and concept-words should be determinate (*NS*, p. 133). But the early Wittgenstein does not recognise this distinction; or if he does, only as a minor, surface-level distinction. He does not judge on the number of logical forms (4.128), and calls them generally "names", which refer to so-to-speak all-purpose objects (3.3411).[1] But if Wittgenstein does not distinguish between objects and concepts, and treats all logical elements as names, the Fregean demand for determinacy, which is formulated for concepts, needs to be reformulated if Wittgenstein wants to keep it. This is what I attempt, in the light of scarce evidence, in section (4.2).

This reformulation attempt, however, has to be understood against the back-drop of how Wittgenstein reinterprets Frege's other demand, the demand for non-empty names. The crucial difference here between Frege and Wittgenstein is that Frege is not interested in a final analysis, while for the early Wittgenstein, this is one of the major motivations. Frege has no qualms to accept *Socrates* as an object, and *wearing a red jacket* as a concept, and leave the analysis at that stage. But for the early Wittgenstein, the fact that these objects (in Wittgenstein's general sense) could not exist is enough for them to fail the demands of his version of the demand for non-empty names (section 4.1).

In section (4.3), I explore the consequences of this interpretation of the *Tractatus* for modern vagueness. Since modern vagueness is a kind of indeterminacy, the *Tractatus* cannot accept it as a feature of ordinary language. Thus, it gets treated the same way that Wittgenstein treats indeterminacy: It is ruled out as a demand of logic. Accordingly, the propositions of ordinary language have a determinate truth-value even if a recalcitrant world seems to threaten this determinacy. But often speakers have no clue about the determinate truth-value, and the task of uncovering it then is delegated to logical analysis. Interestingly enough this amounts to a form of epistemicism: the apparent vagueness of the propositions of ordinary language turns into sharpness if only we *knew* more.

4.1. The picture theory and deep-level analysis

One important contrast between Frege and early Wittgenstein concerns the status of ordinary language. Frege saw a number of defects in ordinary language: ambiguity,

[1] This is a disputed claim. See (Copi 1958) and (Anscombe 1963) for diverging interpretations.

disguised categorical differences, failure of reference and indeterminacy (Glock 2008, p. 30). The first two are notational defects, and a logically perfect notation should avoid them. The other two are serious logical defects in that no proposition which has these defects is a proper, truth-apt proposition. That is why Frege would replace ordinary language with an artificially constructed language such as his *Begriffsschrift* insofar as truth is concerned. Early Wittgenstein concurs with Frege that the ambiguities and category-conflations of ordinary language are a serious impediment for philosophy (*TLP*, pref., 3.323, 4.003), and need therefore to be excluded from a logically perfect notation. Contrary to Frege, the early Wittgenstein insists that ordinary language fulfils the demands of logic and contains no serious logical defects.

> Alle Sätze unserer Umgangssprache sind tatsächlich, so wie sie sind, logisch vollkommen geordnet. (*TLP*, 5.5563)

To be logically completely in order (*logisch vollkommen geordnet*), an ordinary proposition has to be a picture, in the sense of the *Tractarian* picture theory (3, 4). Such a picture is a truth-function of elementary propositions, and elementary propositions are concatenations of names (2.14, 3.21, 5). The meaning of a name is the object it refers to (3.203). The early Wittgenstein relegates the question whether there is a 27-termed relation to the application of logic (4.128, 5.5541). Accordingly, "name" is an umbrella term that covers singular terms, property-predicates, and relation-predicates of any number of terms. One corollary of the picture theory is that to have a sense is equated with having truth-conditions. In contrast to Frege, the *Tractatus* could not say of a sentences-sign such as "Pegasus eats a $\mu\tilde{\omega}\lambda\upsilon$" (cf. p. 58) that it has a sense, but not a truth-value. For the *Tractatus*, for a proposition to have sense is to be truth-apt. Then, the consequence of not fulfilling the demands of logic is, both for Frege and the *Tractatus*, that the alleged proposition is not truth-apt. But the latter draws an even worse morale: Logically defective propositions do not even have a sense; if a proposition is not truth-apt, there is nothing to understand.

However, *prima facie*, it seems that ordinary language *fails* to meet the demands of logic. For one thing, what is the referent of "the present king of France" or of "Pegasus"? For another, is it completely determinate whether a book is green for all shades of bluish green and greenish blue?

The solution to apparent failures of reference of propositions that ought to have sense and be truth-apt is logical analysis. The issue of reference failure arises in a first instance

for expressions that seem to be referring, but turn out to be predicative upon analysis. In the terminology of the *Tractatus*, what seems to be a name turns out to be a description.

2) The present king of France does not exist.

If the expression "the present king of France" is treated as a name, it does not have reference if there is no present king of France. But in the framework of the *Tractatus*, this threatens that (2) has a sense. A proposition has sense only if all of its constituent names have a reference (*TLP*, 3.203, 3.318). Accordingly, (2) could only be true, since in the case of the non-existence of the present king of France, it is nonsensical. This would certainly be absurd – it *is* possible that the present king of France does not exist. The obvious solution is to treat "The present king of France" only as an apparent name. Upon analysis it turns out to be a description. Wittgenstein follows here Russell's treatment of definite descriptions (Russell 1905), which he regards as a paradigm of logical analysis (*TLP*, 4.0031).

While definite descriptions carry their descriptive content on their sleeve, the same is not true for proper names. Nevertheless, in the *Tractarian* framework, proper names such as "Pegasus" or "Socrates" or "Nothung" (a sword of the *Nibelungen*) are then treated in the same manner – as descriptions. The sword Nothung could be shattered into parts and cease to exist.

3) Nothung is in the corner.
4) The blade is in the corner and the hilt is in the corner and the hilt is fixed to the blade.

"Nothung" is not a real name, but turns out, upon analysis, to be a description: (4) is an analysis of (3). Of course, the problem of reference-failure recurs. "The blade" or "the hilt" could equally not exist. In other words, reference-failure is a systematic problem of the view that the meaning of a component of a proposition (a name) is its reference. Here is how early Wittgenstein tries to solve this: He accepts that there are objects that could not fail to exist – he calls them "simple objects" in the *Tractatus* (2.02), and "indestructible" objects in the *Investigations* (*PI*, § 55, 56, 58).

Thus, the threat of reference failure leads the *Tractatus* on a quest for indestructible objects, which will only be revealed by deep-level analysis. In the *Tractatus*, there is a very condensed argument to the effect that objects must be simple (indestructible)

(2.02–2.0212, see also 5.123). How exactly the argument proceeds is highly disputed (see, for instance, White 1972, Ludwig 1976, Schroeder 2006, ch. 2.3, Proops 2011, M. Morris 2016). I follow Schroeder in that I regard the referential doctrine (as part of the picture theory) as the main driving force of the argument, and Proops in that I regard the determinacy of sense as another main component.

If the picture theory is true, then ordinary propositions have the form of a truth-function of elementary propositions, which in turn are concatenations of names. Assume that

Assumption: Objects may be composite.

Since names refer to objects, there may now be a name that refers to a complex object. We can take as our example proposition (3), where "Nothung" is a name for a complex object. According to the picture theory, if a name does not refer to an object, the proposition of which it is part does not have sense (does not have a truth-value). Since this only occurs if the complex object is in fact dissected (and does not exist), we can formulate a dependency relation. For our example, this means that (3) would have sense only if (4) is true. This is reflected in (2.0211).

> Hätte die Welt keine Substanz, so würde, ob ein Satz Sinn hat, davon abhängen, ob ein anderer Satz wahr ist. (*TLP*, 2.0212)

With substance Wittgenstein means here the totality of objects that is common to all possible worlds (2.021, 2.022, 2.023). If there were no simple objects, then the situation described above holds: Whether one proposition has sense depends on whether another is true. But according to the demand for determinacy as I formulate it in section (4.2), a proposition needs to be determinate no matter in what state the world is. This rules out such dependency relations. Then, we can discard in a *reductio* our initial assumption. Therefore, objects must be simple. In other words, determinacy together with the picture theory leads to the claim that objects must be simple.

These simple objects are not known to us yet, but will be revealed by deep-level analysis. Three features of this deep-level analysis are relevant. The first is that each proposition has only one correct and complete analysis (3.25). All logical relations of ordinary propositions can be traced to logical relations between combinations of names. If more than one analysis would be correct, the proposed elementary propositions of

one analysis would not independent of those of the other analysis. But elementary propositions should be independent (1.21, 2.061, 4.211), and the simple objects referred to by them should be the building blocks of reality and common to all possible worlds (2.022).

The second feature is that the true logical structure of propositions is unknown to us, and it is not clear whether human beings have the ability ever to discover it. Remarkably, Wittgenstein doesn't give a single example of a simple object. Moreover, Wittgenstein emphasises that ordinary propositions do not carry their logical structure on their sleeves.

> Es ist menschenunmöglich, die Sprachlogik aus ihr unmittelbar zu entnehmen. ... Die stillschweigenden Abmachungen zum Verständnis der Umgangssprache sind enorm kompliziert. (*TLP*, 4.002)

For Wittgenstein, the true logical structure of a proposition is disguised by its surface. For instance, in the proposition "Green is green," (cf. *TLP*, 3.323) the two occurrences of the same sign have a different logical structure. The first occurrence is a proper name, and the second occurrence is an predicate. But this is only an innocuous example of how the surface of a proposition hides or disguises its true logical structure. The tacit and unknown conventions of ordinary language are of enormous complexity. That is, the disguise is not only about a proper name pretending to be a predicate, but about a much more complex logical structure hiding in a simple-looking proposition. For instance, the deep-level analysis of the proposition "there is a teabag in that cup" should involve descriptions of the string, the label, the clip, the bag and the tea leaves, including descriptions of their individual sizes and shapes, and their relative spatial positions (the deep-level analysis of that proposition should include the items above, but also more than that: tea leaves are also composed of parts). Moreover, it should also say something about what counts as there being a tea bag: How many leaves it takes, and their possible spatial arrangements.

At this point, it is helpful to have a look at the *Notebooks*:

> Es ist also auch *dem* UNBEFANGENEN *Geist* klar, daß der Sinn des Satzes "die Uhr liegt auf dem Tisch" komplizierter ist als der Satz selbst. (*NB*, p. 69)

The terminology of the *Notebooks* differs from the one of the *Tractatus*. It distinguishes between a proposition and its sense, where the sense can be and is often more complicated than the proposition itself. The prime example for this can be found in (*NB*, p. 65):

5) The colour patch is to the right of a line.

Because the colour patch is, at least potentially, made of infinitely many points, potentially infinitely propositions follow from it. But this terminology is not taken over into the *Tractatus*. The proposition itself is a truth-function of elementary propositions, and the difference between the proposition and its sense would threaten the logical independence of elementary propositions. But perhaps one could say in line with the *Tractatus* that an analysis of a proposition is less complex than a deep-level analysis of it. First-level analysis is a logical analysis that follows closely the (school)-grammatical structure of the proposition. For instance, in "Socrates is mortal", we can identify the grammatical subject "Socrates" and the grammatical compound predicate "is mortal". Accordingly, its first-level analysis has the form *f(a)*, where f(x) is a logical predicate and *a* is a singular term. However, the first-level analysis need not follow the grammatical analysis as closely, and there may be mistakes even in the first-level analysis. An example of such a type of mistake might be to analyse the grammatical predicate "moves" as an monadic logical predicate, when in fact it is a logical relation (at least Frege argues in *Über das Trägheitsgesetz* to that effect). The first-level analysis is set in familiar vocabulary with few modifications from the grammatical structure, whereas deep-level analysis is set in unfamiliar or even unknown vocabulary.

The third feature of deep-level analysis pertains to its methodological role.

> Das Resultat der Philosophie sind nicht "philosophische Sätze", sondern das Klarwerden von Sätzen. Die Philosophie soll die Gedanken, die sonst, gleichsam, trübe und verschwommen sind, klar machen und scharf abgrenzen. (*TLP*, 4.112)

This insinuates that logical analysis is to play an important role in philosophy. For our propositions are not clear to us – their deeper logical structure is unknown, let alone their deep-level analysis. What can clear them up is logical analysis. Though logical analysis is, even for the *Tractatus*, not an end, but only a means to resolve philosophical problems (cf. *TLP*, 6.53). This analysis should not only reveal the deeper logical structure of a problematic propositions, but in the process delimit sharply what these propositions say. I take this to be the *Tractarian* adherence to Frege's demand for the sharpness of concepts, which leads us to the issue of determinacy of sense.

4.2. Determinacy of sense

Wittgenstein takes over from Frege the demand for determinacy, but he has to reformulate it since Frege formulates it for concepts, and the distinction between concepts and objects plays no major role for the early Wittgenstein. For Frege, it is a demand of logic that concepts be sharp.

Fregean Indeterminacy: A concept F is indeterminate or unsharp iff, there is at least one object a for which it is neither determinate that a is F nor determinate that a is not F.

Frege's demand for determinacy excludes not only borderline cases of modern vagueness, shades of colour for which it is unclear whether they are green or blue. It also excludes any concept which is is only partially defined, such as Fine's nice numbers (Fine 1975, p. 270).

The demand for determinacy can also be formulated for names. As far as I know, there is no evidence that Wittgenstein did this, but he certainly had to deal with the elimination of Fregean concepts from logical theory.

Name Determinacy: A name N is either concatenated with a concatenated group of names Γ or it is not.

In an elementary proposition, the number of names which are therein concatenated can vary, perhaps, there could even be an infinity of names (4.2211). An object referred to by an name cannot be conjoined with every other object. That an object can be conjoined with another object is part of its nature (2.011). The same holds for names: That a name can be concatenated with another name is part of its nature, which mirrors the nature of the object it refers to. Since more than a pair of names can be concatenated in an elementary proposition, I speak here of a concatenated group of objects (with an unspecified number of objects). They are what remains if one name is removed from an elementary proposition (which is a concatenation of names). In that way, Fregean determinacy of concepts could be generalised to determinacy of names.

A different route to generalise Fregean indeterminacy is via the Law of Excluded Middle or via Bivalence. Frege often equated the demand for determinacy with the *Satz vom ausgeschlossenen Dritten* (Law of Excluded Middle) (*GLA*, p. 168; *GGA*, Vol. II, § 56). Every proposition has to be true, or false, and nothing else. Indeed, if

it is indeterminate whether *a* is F, then the proposition "F(a)" has no determinate truth-value, thereby violating the *Satz vom ausgeschlossenen Dritten*. This indicates in what direction Wittgenstein's generalisation of Frege's demand for determinacy goes. In modern terminology, which distinguishes a purely syntactical Law of Excluded Middle (p or not-p) from Bivalence, the demand for determinacy of propositions amounts to a version of Bivalence.

Propositional Determinacy: A proposition is either true or false (has a truth-value) in any possible state of the world.

Bivalence: A proposition is either true or false.

The only difference is that the demand for propositional determinacy mentions every possible state of the world. But this does not amount to a genuine difference, because it is implicitly included in Bivalence. Statements can not only be made about what actually is the case, but also about what is not the case but is possible. In that vein, if a statement about a non-actual state of affairs would be neither true nor false, this would be a counterinstance to Bivalence.

There is evidence that the early Wittgenstein endorses determinacy for propositions. For one thing, he endorses the Bivalence in the *Tractatus* (4.023). For another, there is also evidence in the *Notes in Logic* and *Notes dictated by Moore* that he endorses the stronger principle of *Bipolarity* that every proposition is capable of being true *and* capable of being false (Glock 1996a, pp. 63f, *NL*, p. 94–99, *NM*, p. 113). Moreover, *Bipolarity* which is another name for a proposition being contingent, follows from the general form of a proposition and the demand that elementary propositions be contingent (4.211). Finally, in the *Tractatus*, Wittgenstein speaks of "determinacy of sense". Since sense pertains to propositions, and to have a sense is to be truth-apt, this speaks for ascribing propositional determinacy to the *Tractatus*.

The perhaps strongest support can be found in the *Notebooks*.

> Wenn der Satz "das Buch liegt auf dem Tisch" einen klaren Sinn hat, dann muß ich, was immer auch *der Fall ist*, sagen können, ob der Satz wahr oder falsch ist. Es könnten aber sehr wohl *Fälle* eintreten, in welchen ich nicht ohne weiteres sagen könnte, ob das Buch noch "auf dem Tisch liegend" zu nennen ist. Also? (*NB*, p. 67)

Here we find the crucial phrase "whatever is the case" (*was immer auch der Fall ist*), and in effect a formulation of what I have called "propositional determinacy". The second sentence, however, shows that Wittgenstein struggled with this demand, because there seem to be undecidable cases. I discuss Wittgenstein's worry that this has the corollary that speakers do not know what they mean in section (4.3).

We have seen in section (4.1) that the picture theory together with determinacy (in particular propositional determinacy) implies that objects are indestructible. Since names and objects are two sides of the same coin, we can take it as well to imply that names cannot be analysed further. This reveals that determinacy plus picture theory has an important consequence for propositions. It is one thing that they must be determinate in all states of the world. But it is another that they themselves must reach down to the deep level of simple objects. Namely, they must be analysable into absolutely simple terms (truth-functions of concatenations of names). That means that a humdrum proposition in some way must say something about yet unknown simple objects. Consider the following example.

6) The Rubick's cube is on the table.

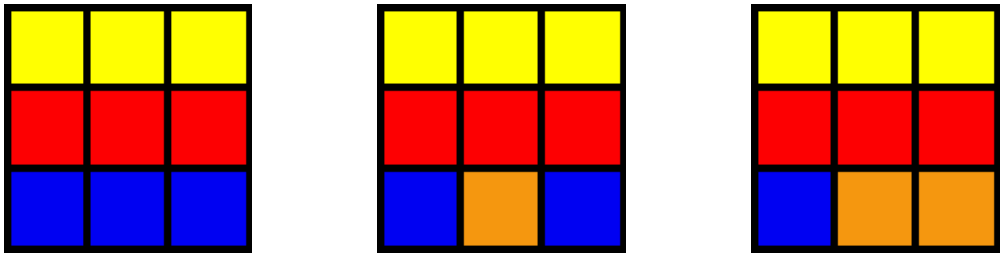
For this proposition to be completely determinate, it must say, among other things: i) Which are the possible locations of the cube for it to be on the table. ii) What possible combinations of colours show on the south side. A strongly simplified way to deal with these issues is shown in figures (1) and (2).

Figure 1.: The cube is at one of the following locations for it to be on the table:

1	2	3	4
5	6	7	8
9	10	11	12

The proposition (6) is unspecific regarding the location of the cube; the cube's being located in more than one position would make (6) true. Accordingly, the predicate "is on the table" can be analysed into a disjunction of the twelve positions above (assuming that these positions exhaust the range of possible locations that make the proposition true). Being in any of those nine positions is a truth-ground for (6), that is a truth-condition which makes (6) true (*TLP*, 5.101). Regarding (ii), matters are a bit more complicated.

Figure 2.: The cube can show any of the following colour combinations (and others) on its south side for it to be a Rubick's cube:



Assume that the simple objects which make up a Rubick's cube are 26 surface tiles and one center tile. Clearly, (6) does not imply that the tiles are coordinated in any particular way. Rather, it leaves open in which way the tiles are combined. But, if the tiles are the simple objects of the cube, the proposition has to say something about their coordination. Is tile 1 adjacent to tile 2 or to tile 3, etc.? Therefore, a complete description of the Rubick's cube that remains unspecific on the combination of the tiles will have to specify a range of possible combinations. This disjunction then implies that a number of colour-combinations could be on the south side. Moreover, if the tiles are themselves logical complexes, the analysis of (6) becomes more complicated, not less.

Whether a proposition is unspecific according to the definition of p. 62 depends, among other things, on what we regard as a "situation". In what terms are situations formulated? The early Wittgenstein's answer is that descriptions of situations are to be formulated in absolutely simple, unanalysable, terms. Therefore, even a proposition like "Socrates is mortal" turns out to be unspecific, since if the situations are described in simple terms, this proposition is true in more than one situation. According to the *Tractarian* demand for determinacy, while the unspecificity of a proposition does not need to be excluded, it has to be transformed. The proposition does not need to specify, for instance, a particular arrangement of colour tiles on the south side, but rather needs to say under which range of combinations the proposition is true. Consequently, the proposition (6) can remain unspecific, but it has to be determinately unspecific.

In the *Tractatus*, the demand for determinacy is expressed in the following, laconic way:

> Die Forderung der Möglichkeit der einfachen Zeichen ist die Forderung der Bestimmtheit des Sinnes. (*TLP*, 3.23)

Wittgenstein equates here the demand for the determinacy of sense with the demand for the possibility of simple signs. For one thing, it is curious that Wittgenstein speaks here of signs and not of symbols, since he certainly has in mind the simplicity that is the end result of an *analysis*. It does not matter at all how many parts a sign has for its logical structure. So we can replace "simple sign" by "simple symbol". What is the possibility of simple symbols? That there are simple symbols? But the *Tractatus'* adherence to the context-principle makes clear that symbols have a meaning only if they are employed in propositions (*TLP*, 3.3). Accordingly, the expression "the possibility of the simple signs" should be read as the possibility that simple symbols can be reached in the analysis of a proposition.

The curious part about this passage is that it identifies the demand for determinacy with the demand for simple symbols. Substituting my interpretations, this gives: The demand that simple symbols be found in analysis is equivalent to the demand that a proposition should be either true or false no matter what is the case. This equivalence claim is less than obvious. But if one can derive one demand form the other (and supporting premises), and vice versa, it would be justified. As I have shown in section (4.1), the demand for propositional determinacy together with the picture theory yields the demand for simple objects. How do we get from here to the demand for simple names? That, I think, is not difficult. In the *Tractatus*, objects and names are structurally analogous elements of atomic facts (*Sachverhalte*) and elementary propositions. What holds for one, holds in structurally analogous ways for the other. Indeed, in the *Notebooks*, there is even the corresponding equivalence claim for objects.

> Die Forderung der einfachen Dinge *ist* die Forderung der Bestimmtheit des Sinnes. (*NB*, p. 63)

That would establish half of the deal. If language is (propositionally) determinate, then its analysis ends in simple symbols (names).

Can we simply turn the argument on its head? Do the picture theory and the postulate that names are simple imply that language is determinate? One might thinks so because it works in one class of cases. According to the picture theory, elementary propositions are concatenations of names. If we are guaranteed that these names refer to simple objects, there is no threat of decomposition. Therefore, there is no indeterminacy due to the dissection of objects. However, could it not be that it is indeterminate whether a name is concatenated with another name? Whether an object is conjoined with another

object? Unless we can exclude this kind of indeterminacy, the reverse argument does not go through. And I don't see that the *Tractarian* theory excludes it. True, that an object can be part of an atomic fact (*Sachverhalt*) lies in the form of the object. But that implies nothing about whether it is in fact conjoined or not, and whether it is determinately conjoined. If this is right, Wittgenstein's equivalence claim appears to be a simple mistake of overestimating the logical relations within his system.

This is my interpretation of (*TLP*, 3.23) in the light of larger issues of interpretation. However, an interpretative puzzle arises concerning the very next remark.

> Daß ein Satzelement einen Komplex bezeichnet, kann man aus einer Unbestimmtheit in den Sätzen sehen, worin es vorkommt. Wir *wissen*, durch diesen Satz ist noch nicht alles bestimmt. (Die Allgemeinheitsbezeichnung *enthält* ja ein Urbild.) (*TLP*, 3.24)

The difficulty is the following: The passage says that a proposition that has an element that signifies a complex is indeterminate. According to *Tractarian* deep-level analysis many if not all propositions of ordinary language have an element (but not a name) that signifies a complex. Thus, most propositions of ordinary language would be indeterminate, and bereft of sense. One way to interpret this would be to say that there are two senses of "determinacy". The first is propositional determinacy, which must be given for a proposition to have sense at all. The second is unspecificity: If a proposition is at least partially unspecific, it says that a determinate range of possibilities is the case. Then, a proposition like (6) would be unspecific, because it does not specify which colour tiles of the Rubick's cube face the south side. But it would be determinately unspecific, because it says which combinations of colours could possibly face the south side. I will not argue against this interpretation, and present instead an alternative interpretation.

My alternative interpretation is encouraged by the ancestor of this paragraph in the *Notebooks*, (*NB*, p. 69). In that period of the *Notebook*, the terminology mentioned above is being used: That the sense of a proposition is much more complex than the proposition itself. And this terminology seems to linger on in the *Tractarian* passage: Not everything is determined by the proposition alone. So the proposition (6) is indeterminate because it alone does not specify the deep level. Only its sense, to be revealed by deep-level analysis, makes sure that the proposition has a determinate sense at all. This lapse into the terminology of the *Notebooks* is curious. It does not cohere with the later assertion that a proposition is a truth-function of logically independent, elementary

propositions. So perhaps this is an indication that the *Tractatus*, while completed aesthetically, is not completed philosophically. Nevertheless, this problem can be solved easily by using a slightly different terminology. The first-level analysis of proposition (6) is indeterminate, whereas its deep-level analysis (and with this, the proposition itself) is completely determinate (even though it can leave some things unspecified).

4.3. Epistemicism *avant la lettre*

Deep-level analysis and the demand for propositional determinacy pave the way for a form of epistemicism. The boundaries of concepts are sharp, but we don't know where these boundaries are (though analysis may reveal them). They are sharp because they are sharp at the deep level, and we don't know where these boundaries lie, because we don't know the deep-level analysis of any proposition. Note that this form of epistemicism is one which in principle appears to be curable, even if the task is a long and arduous one. An epistemic position on vagueness need not decide whether knowledge in borderline cases is in principle impossible, or whether it is just very hard to attain. Williamson, too, leaves the question open (Williamson 1994, p. 201). Note also that Wittgenstein, in contrast to Williamson, does not give any detailed epistemic story why speakers lack knowledge in borderline cases. The two passages considered in the last section are the most pertinent ones in the *Tractatus* for determinacy and modern vagueness. They don't say very much about both issues. Therefore, it is helpful to consider a rather long passage from the *Notebooks*. The passage is from a remark from the 22th June 1915. The date is remarkable as the entry is the climax of two and a half months of thinking about deep-level analysis, complexes and indeterminacy. The entry marks the endpoint of that period of thinking, and also of the physical notebook MS 102. The next remark in a notebook is eight months later. It is philologically plausible that no notebooks are missing from in between (Schulte/McGuinness 1989, p. XVII). Since there are not many remarks about these issues in the later notebook MS 103, we can safely assume that this passage is the last documented, *pre-Tractarian* view on indeterminacy. Therefore, it is not implausible to ascribe some of these views to the *Tractatus*, too. Nevertheless, a word of caution is in order. The *Notebooks* are difficult to interpret not least because Wittgenstein changes his opinion several times, especially in the discussion regarding determinacy and simple objects. The view which I am going to discuss is only expressed on the last two notes of MS 102. Therefore, one can find support for ascribing different

views to Wittgenstein. However, the view I discuss has the advantage of being his last (pre-tractarian) word on the matter.

> [a] Die Abmachungen unserer Sprache sind außerordentlich kompliziert. Es wird enorm viel zu jedem Satz dazugedacht, was nicht gesagt wird. [...]
>
> [b] Ich will nur die Vagheit der gewöhnlichen Sätze rechtfertigen, denn sie *läßt* sich rechtfertigen.
>
> [c] Es ist klar: *Ich weiss*, was ich mit dem vagen Satz *meine*. Nun versteht es aber ein Anderer nicht und sagt "ja aber wenn du das meinst, hättest du – das und das – dazu setzen müssen"; und nun wird es noch Einer nicht verstehen und den Satz noch ausführlicher verlangen. Ich werde dann antworten: Ja, DAS versteht sich *doch von* SELBST.
>
> [d] Sage ich jemandem "die Uhr liegt auf dem Tisch", und nun sagt er "ja aber wenn die Uhr so und so läge, würdest du da auch noch sagen, 'sie liegt auf dem Tisch' ". Und ich würde unsicher. Das zeigt, daß ich nicht wußte, was ich mit dem "liegen" *im Allgemeinen* meinte. Wenn man mich so in die Enge triebe, um mir zu zeigen, daß ich nicht wisse, was ich meine, würde ich sagen: "*Ich weiss*, was ich meine; ich meine eben DAS" und würde dabei etwa auf den betreffenden Komplex mit dem Finger zeigen. Und in diesem Komplex habe ich nun tatsächlich die zwei Gegenstände in einer Relation. — Das heißt aber *wirklich* nur: Die Tatsache läßt sich IRGENDWIE auch durch diese Form abbilden. (*NB*, p. 70)

When Wittgenstein uses the word "vague", or "vagueness", he, just like Frege, does not use that word in the same way as it is used in the contemporary discussion (see chapter 3). In contrast, vagueness here marks out the threat that a proposition might have no sense even though it looks as if it had sense. This threat is prompted if a proposition does not say anything about the deep level. Moreover, it seems that ordinary propositions do not say anything about the deep level. They mention watches and tables, but these are not simple objects. Then, the lack of an articulated deep level is a systematic threat to ordinary propositions. And this lack of an articulated deep level is what Wittgenstein wants to justify. The obvious justification is that the proposition (with no articulated deep level) is connected (via definitions) with an articulated deep level in its sense. Therefore, ordinary propositions are not bereft of sense. But this raises another problem. On the one hand, the deep level is in a way radically unknown to speakers. They are not

in a position to even give a list of simple objects that feature in their proposition. On the other, Wittgenstein wants to maintain that speakers do know what they mean with their propositions, or, we might say, they should know what the propositions they use mean.

In paragraph (d) of the quoted passage, a case comes up which evokes modern vagueness, because a progression and transition in a similarity dimension is involved. Wittgenstein discusses the proposition:

7) The watch lies on the table.

and asks whether he should *still* say that it lies on the table, presumably after it has been moved slightly. In such a case, a speaker might hesitate and become uncertain. This again is an objection against the view that speakers know what their propositions mean. For their propositions are sharp, but they hesitate and are evidently not able to locate the sharp boundary.

These two problems cover two aspects of knowing what one's proposition means. The inability to say anything worthwhile about the deep level is the inability to *explain* what one's words and utterances mean. The inability to consistently draw the sharp boundaries of concepts is an inability to correctly apply a concept in epistemically favourable circumstances. The watch and the table are in plain sight, but the speaker is not able to give a clear verdict on (7).

Given the discrepancy between the demands for determinacy and these inabilities, both to explain and to apply correctly, how can Wittgenstein maintain that speakers know what their propositions mean? He attempts to answer this challenge by a two-pronged strategy. The first part is that speakers are able to add further explanations for particular challenges. In the little dialogue of paragraph (c), Wittgenstein replies with characteristic chutzpah that these things can be taken for granted. Adding some details, we may re-enact the dialogue.

8) A: "The brush is in the corner."
9) B: "I don't understand. I only see bristles and a wood.
10) A: "Bristles that are attached vertically to a wood are a brush."
11) B: "But you haven't said anything about the bristles; that they are vertically attached to the wood."
12) A: "Well, that is something that can be taken for granted."

In the terminology of the *Notebooks*, the proposition uttered in (8) doesn't say that the bristles of the brush are vertically attached to the wood, but its sense does. This is something that a speaker could have added to her utterance after being confronted with a particular situation, if there was some unclarity concerning that issue. One might say here that her knowledge of what her proposition means outstrips the first-level analysis of that proposition. Perhaps, the hope is, one might reach the deep level eventually on that path.

The second strategy claims that even if that hope never is fulfilled, speakers do know at least *something* about the structure of their propositions. Speakers know the first-level analysis of their propositions, but not the deep-level analysis. Nevertheless, since the deep-level analysis has to follow the structure of the first-level analysis, speakers know at least certain structural aspects of the deep-level analysis. The basis for this, in paragraph (d), can be easily misunderstood. The reaction that Wittgenstein describes is that the person points to the watch lying on the table, that is, at a complex, and says: "I mean this." Williamson (1994, p. 281) thinks that Wittgenstein's reply here is that the proposition says that this watch is in exactly this relation to this table. That would be absurd, because propositions such as (7) are not that specific.

The last sentence of the passage says that this fact (the watch lying on the table) can somehow be represented by the form of proposition (7). In the present case, the first-level analysis features the relation of lying on, the predicate of being a watch, and the predicate of being a table. Of course, the real or deep-level structure of that proposition is very different. However, since the first-level analysis is linked to the deep-level analysis by tacit conventions (*TLP*, 3.24d, 4.002), there is a link between knowledge of the first and of the deep level. The deep level has to accord to the first level. Thus, knowledge of the first level will not be effected by deep-level discoveries. Accordingly, speakers have some knowledge of the deep level, but not enough to decide borderline cases.

As it turns out, the first-level analysis plays quite an important role for my interpretation. Knowledge of it is supposed to provide the speaker with stable knowledge of some aspects of the deep level. Can it play such an important role? There is good evidence to think that it does: Wittgenstein considers a problem about our knowledge of the validity of arguments. Take one of the paradigmatic examples of a logical inference. *Socrates is a human being. All human beings are mortal. Therefore, Socrates is mortal.* If we know anything about logic, we know for certain that this inference is valid. However, given the *Tractarian* views on deep-level analysis, there is a problem. Since Socrates and the

properties of being mortal and of being human are logically complex, we don't know the true logical structure of this inference. Wittgenstein's solution is to say that we know something about the form (in the sense of possibility of structure (*TLP*, 2.033)) of the deep-level structure: it has to accord to the structure of the first level (cf. (*NB*, p. 69); Sullivan 2003). This reason for taking the first-level analysis seriously is reflected by a number of ideas and by differences in terminology of the *Notebooks* and the *Tractatus*. For one thing, the last note of MS 102 (*NB*, p. 69–71) endorses a terminology, where a name can refer to a composite ("zusammengesetzt") object.

> Der Name faßt seine ganze komplexe Bedeutung in Eins zusammen. (*NB*, p. 71)

This terminology clearly differs from the *Tractarian* one, where names are simple signs (*TLP*, 3.202, 3.26). It shows that Wittgenstein took first-level analysis very seriously. This terminology even lingers on in the *Tractatus*:

> Die Zusammenfassung des Symbols eines Komplexes in ein einfaches Symbol kann durch eine Definition ausgedrückt werden. (*TLP*, 3.24)

Here, we have a similar formulation and a similar puzzlement. For this passage says that the symbol of a complex can be contracted into a simple symbol. So again, simple symbols can hide logical complexity. My best explanation for this is that Wittgenstein is operating with two senses of "simplicity": First, the logical simplicity of the first-level analysis, which ensures our knowledge of some types of inferences. Second, the simplicity of simple objects, which ensures that ordinary propositions have sense. In any event, these considerations suggest that Wittgenstein indeed thought that the first-level analysis plays an important role in our knowledge of the logical form of our propositions.[2]

[2]It is noteworthy that the most important ideas of *TLP* (2.02f) were first formulated in the *Proto-Tractatus*. So here is my preferred interpretation: At the end of MS 102 (the *Notebooks*), Wittgenstein has come to the view that the sense of a proposition is more complex than the proposition itself. This addresses the systematic problem of reference failure. At the same time, worries about our knowledge of the validity of inferences leads him to speak of names for composite objects. However, in the *Proto-Tractatus* and *Tractatus*, names are reserved for symbols with no logical complexity. The topic of our knowledge of inferences drops out of the picture and is only touched upon in the obscure remark (3.24d). Nevertheless, the new material of the ontology of *Proto-Tractatus* and *Tractatus* and especially the (2.02f) as well as (3.202, 3.26, 4.002) make it clear that there is a deep level of simple objects with which speakers are unacquainted.

Let us return now to the long quotation of the *Notebooks* quoted at the beginning of this section. Could this passage be read in a way which does not support epistemicism? My interpretation above supports the ascription of epistemicism to early Wittgenstein, because it in effect states that speakers lack knowledge of borderline cases. The other part of epistemicism, that there is no indeterminacy in borderline cases in language, is not supported directly by that passage, but by my deliberations about determinacy in the last section. However, one could attack the ascription of epistemicism to early Wittgenstein by providing an alternative interpretation of the quoted passage above which does not subscribe a lack of knowledge about what they mean to speakers. Speakers obviously don't know the deep-level analyses of their propositions. But perhaps one could say that they know what they mean independently of knowledge of the deep level. According to that interpretation, the *Tractatus* would hold that language is determinate, and that speakers know how it is determinate, that is, where the boundaries of concepts are. There are three ways to fill in the blanks. The first way is to insist that speakers simply know the boundaries. But this flies in the face of well known facts and of the passage we just discussed: Sometimes speakers hesitate in borderline cases. The second alternative is that the speaker can authoritatively stipulate what he means and what he means to exclude. If he says in borderline cases that a is F, then a is F. In that way, the sharp boundaries of concepts would be determined on the go, and, in borderline cases, speakers would have to find out whether their drawing of the boundary corresponds with each other, or they would have to agree on a common stipulation. This, however, does not fit the passage above. For the speaker does not stipulate boldly new boundaries, but instead becomes uncertain. The third way is to claim that what speakers mean is very specific. If they say that a Rubick's cube is on the table, they mean for instance that it is at position one, and that its south side has the first colour combination (plus some other specifications). If what speakers said were always as specific, borderline cases could never arise. However, the cost of high specificity is that the proposition can be easily falsified, and will only be true in very special circumstances. – But this seems to be a truly awkward analysis of our ordinary propositions. They simply are not that specific and prone to falsification. Moreover, even if the specification strategy worked for utterances concerning present things and people, it is hopeless for speaking about absent things. "Yes, I have a frying pan in my kitchen" does not say anything about the size and colour of the pan. Thus, these alternative readings are no serious competitors for my interpretation.

If my interpretation of this passage is correct, and if it also reflects the tacit views

of the *Tractatus*, then the early Wittgenstein thinks that ordinary propositions have a highly complex deep-level structure, and that speakers know something about this deep level. Nevertheless, their knowledge has limits. Borderline cases are an instance of such a limitation of knowledge. In such cases, it is unknown and has to be discovered what the speaker means in full determinate detail. In other words, the process of deep-level analysis is supposed to resolve borderline cases. This, of course, is reminiscent of Williamson's theory of vagueness. Williamson trades on a specific relation between meaning and use, and holds that there are different but very similar meanings, such as *heap*$^{\circ}$ and *heap**. Both are precise, but according to one of them the minimum number of grains required to form a heap is 43, according to the other it is 42. Williamson remains radically unspecific as to what we need to know about use in order to locate the sharp boundaries (Williamson 1994, ch. 7, Wyss 2010, ch. 2). – But then, how could we ever know where they are? Similarly, Wittgenstein remains unspecific on how the deep-level structure of a proposition can be revealed. It is the application of logic that is supposed to achieve this feat (*TLP*, 5.5541, 5.557). But on the basis of what are we to apply logic? There is a possible answer to that: the *Tractatus* sees a connection between meaning and use.

> Um das Symbol am Zeichen zu erkennen, muß man auf den sinnvollen Gebrauch achten. (*TLP*, 3.326)

So perhaps, use could serve as a guide toward the hidden, sharp meaning? There are both similarities and dissimilarities between the *Tractatus* and the *Investigations* regarding meaning and use. I take it that the same supervenience relation holds for both early and later Wittgenstein: Meaning supervenes on use. If meaning is different, use is different, too. At least that would explain how use can serve as a guide towards meaning. One difference is, however, that, for the *Tractatus*, what it makes sense to say is itself determined by the simple objects (cf. Glock 1996a, p. 376). In contrast, for the *Investigations*, there are no concepts which are independent of linguistic use. Nevertheless, for the *Tractatus*, knowledge of the use of an expression is the correct route for knowledge of its meaning. In the case of unspecificity, this is halfway plausible, at least in the framework of the *Tractatus*. Material parts are logical parts, too, since they could be arranged differently, and objects must be simple. Certain arrangements of parts we call a watch, others not. In this sense, the use of words may reveal which arrangements of watch-parts are watches. However, in the case of unsharp boundaries, it becomes unclear how we could recognise the symbol from its use. Consider again

the *heap*° vs. *heap** case. According to the *Tractatus*, some feature of use should help us decide which of the meanings is the actual meaning of our sign. But there seem to be no features of our use of heap which could decide the matter. On the contrary, in some borderline cases, we become uncertain, as Wittgenstein suggests in his own little dialogue.

To finish, let me draw some support from later writings for ascribing epistemicism to the *Tractatus*. There are two striking passages.

> Und es scheint mir nun, daß diese Verschwommenheit nicht etwas Vorläufiges ist, das genauere Erkenntnis später eliminieren wird, sondern eine charakteristische logische Eigentümlichkeit. (*PB*, p. 263)
>
> Und wissen wir selbst denn mehr? Können wir etwa nur dem Andern nicht genau sagen, was ein Spiel ist? – Aber das ist nicht Unwissenheit. Wir kennen die Grenzen nicht, weil keine gezogen sind. (*PI*, § 69)

Common to both passages is that Wittgenstein explicitly denies the epistemic position for unsharp boundaries. The blurredness of experience, and our inability to locate the boundaries is not a case of ignorance. It is a logical feature of experience that it is blurred, and it is a logical feature of the concept *game* that no boundaries are drawn for it. The passage from the *Remarks* is even more striking, since it begins with "And, now, it seems to me. . . " (*PR*, p. 263). This marks a contrast to an earlier view. And in the case of the *Remarks*, there is some evidence that he is criticising the *Tractatus*. For the *Remarks* are the second even half-complete work after the *Tractatus*. The first is *RLF*, which does not deal with vagueness. Moreover, there are no other views on vagueness in the *Remarks* to which he could be alluding to.

Conclusion

It would be anachronistic to ascribe to the *Tractatus* a theory of modern vagueness. For that, it says too little about the whole issue. Nevertheless, there are two main reasons for ascribing epistemicism about modern vagueness to Wittgenstein. The first is strategic. The *Tractatus* holds that propositions describe a deep level of simple objects which is unknown to us. That is, at least according to my broadly Hackerian interpretation, it endorses what could be called epistemicism for deep-level meaning: Propositions of

ordinary language describe the deep level of simple objects, even though we don't know what they say about it. This is a corollary of the picture theory together with the demand for determinacy, as the early Wittgenstein formulates it. Since Vagueness is a subkind of indeterminacy, it too is excluded and its determinate dissolution is relegated to deep-level analysis. The other reason is exegetical. The long passage from the *Notebooks* shows, according to my interpretation, that Wittgenstein at some point thought about borderline cases: that they are due to a lack of knowledge, and not due to a lack of determinacy in language.

5

The Middle Wittgenstein on Vagueness

Wittgenstein's views on determinacy took a sharp turn from the *Tractatus* to the *Investigations*. In both works, he was interested in understanding the logic or grammar of ordinary language. But in the *Tractatus*, he believed that such an ordinary language had to abide and did abide by certain logical demands; in particular the demand of determinacy: That a proposition is either true or false no matter in what state the world is (see chapter 4). But in the *Investigations*, he came to see this demand both as unnecessary and impossible (see chapter 6). This essay explores his views in the transitional phase in between those works.

After Wittgenstein returned to philosophy in 1929, he tried to reform the system of the *Tractatus*. He had found that there was a problem with the exclusion of colours. The sentence "This patch is red all over and green all over" was not a contradiction that could be rewritten as a logical contradiction "p and not p". Rather, he concluded, it was part of the meaning of colour words such as "red" and "green" that they excluded each other. But then, the elementary propositions of the *Tractatus* would not be independent anymore, and the dream of a purely truth-functional analysis of language collapsed. The rules that governed language turned out to be much more complicated than he had thought at the time of the *Tractatus*.

His first longer work after the break, the *Philosophical Remarks* from 1930, was an attempt to state the rules of language in the way that would continue the project of the *Tractatus*, but take into account the further complexity required by the colour-exclusion problem. The next near finished work, the *Big Typeskript*, however, broke explicitly with the *Tractarian* demand for determinacy of sense and acknowledged that for some cases, language had no rules ready. Both works contain a few remarks on the topic of vagueness,

and I will discuss them in turn. While both do not discuss the sorites paradox, there is reference to "the problem of the sand heap". Neither the *Blue Book* or *Brown Book* or other passages from the *Nachlass* add anything new on the topic of vagueness to what can be found in the the *Philosophical Remarks* and the *Big Typeskript*.

5.1. Blurredness in the *Philosophical Remarks*

The *Philosophische Bemerkungen (Philosophical Remarks)* is the first work of Wittgenstein after the *Tractatus*, which can be called a "book", even if it is not in a finished, publishable state. Many ideas of the *Tractatus* have survived. For instance, Wittgenstein claims that the form of language corresponds to the form of the world (*PR*, p. 85), and he adheres to a model of logical analysis where the final analysis of a proposition differs tremendously from its outward appearance (see the distinction between primary and secondary language, which follows below). Regarding the *Tractatus*, one important aspect of simple objects that is disputed by commentators is the nature of simple objects: Are they material particles or phenomenal *minima sensibilia* or is the issue left open for the application of logic? In the *Philosophical Remarks*, however, Wittgenstein explicitly subscribes to phenomenalism: There is a primary phenomenal language, which refers to what is phenomenal, to immediate experiences. In contrast, the physical language is secondary; its terms are constructed on the basis of terms of the primary language. As a consequence, Wittgenstein is concerned with the logic of the visual field. The idea of a visual field and its own logic is, perhaps, partially motivated by the following observation. If I look out of a window and see a tall tree, it is clear that the tree is larger than the window in the physical world (*PR*, p. 100-1). In contrast, there is another sense in which we can say that the window frame is larger than the tree, or appears larger. And this sense is supposed to be catered for with the idea of a visual field.[1]

The *Remarks* is committed to ideas which Wittgenstein later attacks with the private language elenctic argument. For the primary phenomenal language is a language that can, in principle, be understood by only one person. However, a private language friend, for instance middle Wittgenstein, would not be able to explain to himself what a word "w" of her private language means. At least not, if he does not presuppose an existing

[1] There is an additional complication: Wittgenstein even talks of two different senses of "larger" in visual space. One could try to spell this out as follows: For one thing, the window frame appears larger than the distant tree. For another, it belongs to the perception of the tree as a three-dimensional object that it is larger than the window but more distant.

language. If he explained "w" as the sound that a whisk makes, he would have explained it in public terms, and it would thereby cease to be private term. Moreover, to say that we see things in the visual field is misleading. Consider the case of the straight stick which appears bent when half-immersed in water. In ordinary discourse, it would be uncommon to say that we see a bent stick, because seeing is an achievement verb. We could say that we "see the stick *as* bent" or that the stick "appears bent". But there is not a second sense of *seeing*. Then, the puzzle would be that the tree *is* larger than the window but that it *appears* as being smaller. Wittgenstein's question then turns out to be: Can we describe how the world appears to us? And to this question the answer is "yes", even under the conditions of the private language argument. For instance, the subjective viewpoint is taken into consideration by artists as perspective. In perspective, a pencil's head may appear larger than its tail. And as far as perspective (in the literal sense) is concerned, the laws of optics can be used to describe, explain and predict the perspective of a subject.[2]

In the *Remarks*, the topic of vagueness is discussed in connection with the visual field. Wittgenstein comes to the view that blurredness[3] ("Verschwommenheit") is a logical feature of experience.

> Sobald man exakte Begriffe der Messung auf die unmittelbare Erfahrung anwenden will, stößt man auf eine eigentümliche Verschwommenheit in dieser Erfahrung. D. h. aber nur eine Verschwommenheit relativ zu jenen Maßbegriffen. Und es scheint mir nun, daß diese Verschwommenheit nicht etwas Vorläufiges ist, das genauere Erkenntnis später eliminieren wird, sondern eine charakteristische logische Eigentümlichkeit. Wenn ich z. B. sage: 'ich sehe jetzt einen roten Kreis auf blauem Grund und erinnere mich, einen vor ein

[2] To talk of descriptions of subjective experiences could be seen as going against the grain of the later Wittgenstein who treats an utterance like "my foot hurts" as an avowal. But first, it would not be uncommon for a doctor to ask his patient to describe his pain. The verb "to describe", as it is commonly used, has a wider field of application than some Wittgenstein readers may think. Some applications involve the primacy of a sincere avowal, others not. Secondly, Wittgenstein does not warn against the use of the word "description" for sensations, but a particular picture of it (cf. *PI*, § 290-1).

[3] The translators have here "vagueness". I use the neologism "blurredness" in order to mark that Wittgenstein used a different word, here. There is a related adjective that Wittgenstein uses in *PI*, § 71, 77: "verschwommen" (blurred). He applies it to "border", "concept" and "picture". And apart from an emphasis on the visual, there is no significant difference to "unscharf" or "vague" in the *Investigations*. I don't want to blur differences, even if they turn out to be only on the level of the sign.

> paar Minuten gesehen zu haben, der gleich groß oder vielleicht etwas kleiner war und ein wenig lichter', so ist *diese* Erfahrung nicht exakter zu beschreiben. (*PB*, p. 263)

What is this blurredness? The example that Wittgenstein mentions above is a complex one. The subject compares two experiences, each of a red circle on a blue ground. Comparing the colour and the size of the two circles, the subjects verdict is: "same size or a bit smaller" and "a little lighter". Is Wittgenstein thereby discussing the pitfalls of memory? No. For the timespan between the two experiences is only a few minutes, and the same blurredness would occur if the subject held a piece of paper with a drawing of a circle in each hand. The blurredness is not a blurredness of memory. Rather, what is at issue is that just by looking, we cannot make a more exact comparison. We have experienced the first circle in a certain way, and the same holds for the second circle. The comparison of these experiences yields blurred results, which is not "to be eliminated later on by more precise knowledge" (*PR*, p. 263). Now, the obvious suggestion would be to measure the circles with a ruler, and compare the colours with a colour chart. But Wittgenstein conceives of visual space in radical contrast to physical or Euclidian space.

> Wenn ich sage, man kann ein Tausendeck nicht von einem Kreis unterscheiden, so muß mir hier das Tausendeck durch seine Konstruktion, durch seine Entstehung gegeben sein. Denn, wie wüßte ich sonst, daß es 'tatsächlich' ein Tausendeck ist und *nicht* ein Kreis?
>
> Im Gesichtsraum gibt es keine Messung. (*PB*, p. 266)

Wittgenstein conceives of the visual space or field as one without measurement. Here is my attempt at fleshing it out. When I lay a ruler to a circle, I can measure the physical circle, but not the phenomenal one in my visual field. By bringing my eyes closer to the piece of paper, the circle expands, and with it, the ruler. That is to say that the ruler itself is subject to those changes that Wittgenstein wants to capture with talk of a visual space. Now, in my opinion, these effects can be considered by applying the laws of optics or the art of painting in perspective. The three-dimensional world can be projected onto a two-dimensional plane. And in this projection, it has obvious sense to say that the window frame is larger than the tree, or that an ordinary round coin has an oval shape. Or take another example. When I see a polygon with more than 30 sides, I can count the sides of the physical polygon, and determine how many sides *it* has by writing a number

next to each side. But by doing this, I only count the sides of the physical polygon. As such, this procedure cannot produce a better description of what I *see*. I may be able to judge that it has more than 20 sides, and less than 40. But the best description I can give will remain unspecific for a certain range, and degree vague as to whether the figure has 39 or 40 sides.

Let me now consider two aspects of this blurredness. The first one is brought up by the quotation on p. 87: The blurredness is relative to concepts of exact measurement. I take this to mean that a certain concept F is blurred relative to a class of concepts γ such that for some descriptions of a situation in terms of γ, it is indeterminate whether F applies or not. One example that Wittgenstein mentions is the contrast between what a subjects sees as a circle, "the visual circle", and what can be constructed and measured on a piece of paper, the drawn circle. For every drawn circle, one can find impurities, given one has enough precise methods of measurement. Therefore, no drawn circle has the absolute precision of true geometrical circle (not least because there are construction rules for infinite precision). This is of course the old question of whether mathematical shapes are ever realised in the world (cf. Keil 2010b, p. 66). But for the visual circle, this cannot be said. For the methods of measurement, such as applying a ruler and a magnifying glass, do not apply to the visual circle. Therefore, what we see as a circle corresponds to a class of physical shapes, including for instance a chiliagon. (*PR*, p. 265) Accordingly, Wittgenstein could say that the term "visual circle" is blurred in terms of drawn shapes, that one can describe with the geometrical vocabulary of the physical world. Analogously, visual colour would be blurred in terms of colours of physical bodies. This can be made explicit with a blueprint for a psychological experiment. A subject is asked to say of two red patches whether they are of the same colour. The left one is held constant (without the subject knowing), the right one varies according to a very precise colour chart (which figures here as the exact concept of measurement). The expected outcome is that the subject will say of a vague range of hues of red that they are the same colour as the left one. Therefore, what the subject calls "red" is blurred relative to the colour chart.

Is the relativity, which the middle Wittgenstein claims for blurredness, true of other notions in the discussion on vagueness? For Fregean unsharpness, the answer is no. For a concept F to be unsharp it suffices that it is indeterminate for some object a whether a is F or a is not F. There is no need to mention any other concept than F. For modern vagueness, the answer is yes. Modern vagueness effectively is Fregean unsharpness with

the addition that it is located in a transition zone. And the latter is explained as an underlying dimension δ which is ordered according to similarity or quantity. (Similar considerations hold for the concept of *borderline case*.) Heaps are vague in terms of how many grains they contain, richness is vague in terms of the amount of money owned, the sound of a trumpet is vague either in terms of a qualitative description or of sound wave frequency clusters, red is vague in terms of Goodman shades or in terms of light wave frequencies. The idea that modern vagueness is relative to an underlying dimension is expressed in Wright's and Sainsbury's idea of Tolerance: Large differences in the underlying dimension δ affect whether F applies or not, but small changes in δ do not (Wright 1975, p. 333, Sainsbury 1989, p. 2).

The second aspect is what is today called "higher-order vagueness".

> Und hier stoße ich auf die Hauptschwierigkeit, denn es scheint, als wäre auch die exakte *Begrenzung* der Unexaktheit unmöglich. [...] Wie wenn man einen Sumpf durch eine Mauer abgrenzt, die Mauer ist aber nicht *die* Grenze des Sumpfes, sondern sie steht nur um ihn auf festem Erdreich. Sie ist ein Zeichen dafür, daß innerhalb ihrer ein Sumpf ist, aber nicht, daß der Sumpf genau so groß ist wie die von ihr begrenzte Fläche. (*PB*, p. 264)

When considering the example of a visual circle which corresponds to a class of drawn geometrical figures, Wittgenstein considers how to demarcate that class of geometrical figures. He suggests vibrating a circle, and all geometrical bodies of which all points lie within the thus generated area make up the class of corresponding figures. But he immediately objects that one could also let vibrate a hundred-sided polygon. Therefore, the class of corresponding figures is not complete. Moreover, Wittgenstein argues that this way of producing the corresponding class misses a crucial point of blurredness. If there was a precise demarcation that separates the figures that I see as a circle and those that I see as not being a circle, I would need to be able to reliably notice minute differences, which amounts to the power to see Euclidean lines. But to say this of the visual field makes no sense, since the procedures for determining of a line that it is Euclidean is not available for visual space (*PR*, p. 265). Wittgenstein's swamp analogy is a compelling presentation of this logical feature both for his view in the *Remarks*, but also for any view that subscribes to higher-order vagueness. We can hedge in the borderline cases (from one or from both sides). But the hedge so produced is not the actual separation line between being F and not being F.

Blurredness is clearly concerned with modern vagueness, since Wittgenstein alludes to the inexactness of inexactness, and thereby to higher-order vagueness. Moreover, the way he construes his examples suggests that he is also concerned with unspecificity. There is the visual circle that corresponds to a *class* of physical shapes, and the very first example, where, after comparing two circles, the most exact verdict is "they are of equal size, or one is a bit smaller". In the second case, the experience is blurred also in the respect that the most exact description of it is the logical disjunction above. I present my best attempt at a characterisation of blurredness in the following. The term "correspondings" should be sufficiently clear[4] when considering examples. Visual figures correspond to physical figures, phenomenal colours correspond to "physical" colours.

Blurredness: A phenomenal concept or experience is blurred iff it corresponds to an inexactly bounded class of physical "correspondings".

To finish, let me mention a few peculiarities of that notion. First, blurredness applies only to concepts such as colour words and figure words, where perception is directly involved. Accordingly, such perceptual terms correspond to an inexact class of physical colour shades or figures. But not all concepts are directly involved with perception. Especially social terms such as "rich", "married", "friend" or "money" or (partially) functional terms such as "hammer", "radio" or "chair" fall out of the picture.

Second, as I have mentioned before, the *Remarks* is committed to a phenomenal, private language and incurs the problems that come with it. The term "visual circle" is subject to that tension. Either the methods of measurement of the physical world cannot be applied to the circle as it appears to us. But then, why should we call it a "circle" in the first place? Geometry defines a circle as a set of all points in a plane that have the same distance from a particular point. And this presupposes that the distance between the points can be measured. And redrawing a given circle with a compass presupposes that the compass can be pinned at the centre point. Or they can be applied, and we can describe our visual perspective of the world as either according to what optics would predict or as diverging from it. Optics cannot overrule in all cases our judgements about perspective. Under the influence of a drug, we may perceive extraordinary things. Or there may be an extreme case where no causal condition that would explain a deviant perspective can be found until judgement day, and the person nevertheless assures that

[4] Of course, the idea of physical object that corresponds to a phenomenal one falls victim to the private language argument.

she sees things in an abnormal way while her truthfulness stands beyond doubt. Even in that case, the description of her abnormal experience presupposes for its intelligibility a connection to vocabulary which other's could in principle understand.

Third, the focus on the visual field ties vagueness to the absence of methods of measurement. But here, I and the middle Wittgenstein disagree. Concepts of ordinary language are vague and can potentially be subject to highly exact methods of measurement. The term "circle" for instance, as it is used in ordinary language, is vague and not absolutely precise, even though things we call "circle" can be subjected to state-of-the-art methods of measurement. Thus, it can (and will) turn out that what is called "circle" according to its ordinary meaning, such as one which is produced by compass, has some impurities and therefore does not fulfil the geometrical definition. The difference between an ordinary circle and a geometrically perfect one is that the standards of exactness differ for the two. And these standards vary with the needs and purposes of those who employ the concepts.

5.2. The Problem of the Heap in the *Big Typescript*

The discussion in the *Remarks* about blurredness features a solitary sentence which links blurredness to the problem of the heap: "Das alles hängt mit dem Problem zusammen 'wieviele Sandkörner geben einen Haufen?' " (*PB*, p. 263). While this seems to ascribe an important place to a problem which lies at the heart of the sorites paradox, the idea is not spelled out. The *Big Typeskript* contains an even more important reference to the problem of the heap: One of the titles of its chapters reads: "Der Begriff 'ungefähr'. Problem des 'Sandhaufens'." (*BT*, p. 136). And indeed, here we find a few pages that deal with the grammar or logic of the concept *heap*. However, no explicit examination of the sorites paradox follows. And this is the closest that Wittgenstein gets in the whole *Nachlass* to engaging with it.[5]

The *Big Typeskript* contains *in nuce* many ideas of the *Philosophical Investigations*, including the ideas of family resemblance, language games, and the criticism of adhering

[5]The *Big Typeskript* and the *Philosophical Grammar* contain much of the same material, albeit with important differences, since the editors of the latter took Wittgenstein's reworkings of the former into consideration. However, concerning the chapter on the problem of the heap, the differences are negligible. The editor of *PG*, Rhees, makes a decision on the changes and marginal notes that Wittgenstein wrote into TS 213, whereas Luckhardt's and Aue's edition of the *Big Typeskript* can be regarded as a critical edition of TS 213.

to a unique ideal of exactness. Baker and Hacker write sometimes as if the Wittgenstein of the *Big Typeskript* was still wedded to the idea of language being akin to a calculus (Baker/ Hacker 2009, p. 138), which would, *pace* § 81 of the *Investigations*, exclude indeterminacy. But there is good evidence that the *Big Typeskript* was not committed to the calculus model *in that sense*. First, the idea of comparing language with a calculus can be put to good use, for instance as a reminder that we are not dealing with causal, but with normative questions. Second, the evidence that Baker and Hacker bring forth (a number of passages where the word "calculus" occurs) does not establish that he holds that language is a calculus of exact rules that apply in every situation. Third, the chapter 58 of the *Big Typeskript* directly concerns the issue of a lack of rules and contains ideas that crop up in the *Investigations* again: The rules of Tennis don't legislate the case that the ball spins around the earth; the proper name "Moses" is used without fixed meaning; and the repudiation of the idea that logic is a normative science (*BT*, p. 249, 252–254; see also chapter 6).

In any event, ideas that come up in the *Investigations* shall not be discussed in this chapter. For one thing, it contains them in their most advanced and exegetically authoritative version. For another, the task of spelling out out differences between these ideas in the *Big Typeskript* and in the *Investigations* would require a vast and complex undertaking. Therefore, I will focus on the *Big Typeskript's* chapter on the problem of the heap. One last thing to note is that the *Big Typeskript* contains, comparable to the *Remarks*, allusions to a "visual space", especially in the part entitled "Phänomenologie". In addition, both are earlier than Wittgenstein's criticisms of a private language. I will refrain from comparing the views of the *Big Typeskript* on the blurredness of experience with the views expounded in the last section. In the following, I will reconstruct the views of the *Big Typskript's* section on the problem of the heap by discussing three points: the concept *roughly*, a psychological experiment, and some aspects of making the concept *heap* more precise. The section starts with:

> "Er kam *ungefähr* von dort →."
>
> "Ungefähr *da* ist der hellste Punkt des Horizonts."
>
> "Mach' das Brett ungefähr 2m lang."
>
> Muß ich, um das sagen zu können, Grenzen wissen, die den Spielraum dieser Länge bestimmen? Offenbar nicht. Genügt es nicht z.B. zu sagen: 'der

> Spielraum ± 1 cm ist ohne weiteres erlaubt; ± 2 cm wäre schon zu viel'? – Es ist doch dem Sinn meines Satzes auch wesentlich, daß ich nicht imstande bin, dem Spielraum "genaue" Grenzen zu geben. Kommt das nicht daher, daß der Raum, in dem ich hier arbeite, eine andere Metrik hat, als der Euklidische? (*BT*, p. 136)

The word "roughly" has an adverbial function: it enlarges the margin of tolerance or establishes a margin of tolerance. Importantly, this margin has no fixed limits. This is why this word is helpful for studying vagueness. Compare the two sentences:

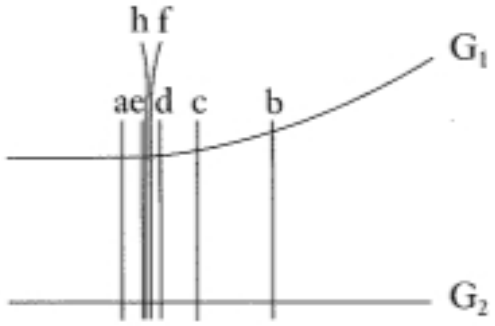
1) Make the board 2 m long.
2) Make the board roughly 2 m long.

While one could perhaps maintain that the order (1) has precise conditions of fulfilment, to say the same for (2) incurs the cost of mounting absurdity. What is required for the sentence to have a good sense that there is a margin, say of ± 1 cm, which is allowed, and another which is too large, say of ± 2 cm. But that doesn't mean that any divergence smaller than 2 cm is allowed, or that the speaker knows where to draw an exact boundary. Even if one forced the speaker to draw a sharp boundary, a new problem would emerge. If one were to try to state exact boundaries of the margin of tolerance, one would soon be entangled in contradictions by saying of one length both that it is allowed, and a later occasion, that it is not allowed (*BT*, p. 136).

The last sentence of the quote above is puzzling. Wittgenstein contrasts the "space in which I am working here" with "Euclidean space". The latter should be familiar by now: It is a space which is governed by the rules of Euclidean geometry: Points have no extension, lines no width, and a tangent of a circle intersects it at exactly one point. In the *Remarks*, as discussed in the last section, the visual space is contrasted to it. There, a circle and its tangent run together for a small distance. But is the "space in which I am working here" referring to visual space? If so, Wittgenstein would have abandoned the view of the *Remarks* that there can be no measurement in visual space, for sentence (2) obviously is about measuring a board (*PB*, p. 266). And there is evidence to that effect (cf. *BT*, p. 449). Moreover, on the next page, Wittgenstein speaks of a "perceived length", which seems to count in favour of visual space. Nevertheless, to speak of a 2 metre long board refers not to its perceived length, but to its length in the material world. So perhaps the "space in which I am working here" does refer to something else than

visual space. My educated guess is that Wittgenstein acknowledges here that ordinary concepts, in a non-phenomenal sense, may be unsharp, referred to by the phrase "space in which I am working here".

So far, Wittgenstein has introduced an adverb whose vagueness is hard to deny. Next, he presents a "psychological experiment" which concerns the concept *same perceived length*, and is thus clearly located in visual space.



The drawing presented here (copied from) *BT*, p. 137 is insofar inaccurate it shows the help lines G_1 and G_2 and more than two vertical lines. In a properly conducted experiment, the subject would be presented with two vertical lines and then asked whether they had the same length. Otherwise, he could infer from looking at the help lines that two given vertical lines cannot have the same length. Wittgenstein imagines that for the lines a and b, the subject replies that he perceives one of them as longer than the other. Then the distance between a and b is halved, and line c drawn. This is repeated until the subject replies that the two lines he now sees are of the same length (in this graph, this is at line e). Then, the distance between the two lines is enlarged again, and the distance between e and d is halved. Now, the subject sees the resulting f as larger than a.[6] That is, we have now found some provisional boundaries. The lines up to e are perceived to have the same length as a. The lines longer than f are perceived to be larger than a. One result is that what is perceived to have the same length as a is not

[6] I think that Wittgenstein made a slip by writing " 'Siehst du f grösser als e?' ", when he should have written "Siehst du f grösser als a?" This assessment is based on the conclusion of the psychological experiment. It's point is to find a vague interval of lines which have the same perceived length as a. Moreover, how could the subject ever perceive f to be longer than e? That is the smallest difference so far considered. The slip is already contained in the ancestor MS 114, p. 7r. Faulkner (2010, p. 167) simply notes that "recall that line e is seen as the same length as the original line a." But in contrast to "is of the same length", "is of the same perceived length" is not transitive. Therefore, the two questions above may very well receive different answers.

one Euclidean length, but a whole interval of Euclidean lengths. A second result is that, according to Wittgenstein, the interval is vaguely delimited. Why? Wittgenstein does not elaborate the point, but my educated guess is that if one pressed further, one would not get a last line that is perceived by the subject to have the same length as a. Rather, the subject would get entangled in contradictions and classify a line once as being of the same length as a, and once as not being of the same length as a. As Wittgenstein remarks a few pages later (*BT*, p. 139), one could devise experiments where one receives a sharp interval as a result. For instance, if, starting from b, lines are drawn ever closer to a, and the first line where the subject replies "same length" is taken as an end point. But that, of course, would measure something entirely different (more on that below).

The upshot of the experiment is that the expression "same perceived length as line a" is unsharp. This has strong affinities with the claim of the *Remarks* that experience is blurred. For, again, a line in visual space is related to an unsharply bounded set lines in Euclidean space: "daß einer gesehen Länge a im euklidischen Raum nicht eine, sondern ein Intervall von Längen entspricht" (*BT*, p. 138).

In the following, Wittgenstein is interested in the fact that we are now dealing with two different concepts of *interval* with differing grammar. For instance, for unsharp intervals, it does not make sense to ask whether they have an endpoint in common, because it is essential for them to have no endpoints. Next, he sketches a similar contrast for concepts that are connected to measurement (and margins of tolerance).

> So könnten wir natürlich auch ein Wägungsresultat "das Gewicht eines Körpers" nennen und es gäbe dann in diesem Sinn eine absolut genaue Wägung, d. i. eine, deren Resultat nicht die Form "G ± g" hat. Wir haben damit unsere Ausdrucksweise geändert, und müssen nun sagen, daß das Gewicht des Körpers schwankt und zwar nach einem uns unbekannten Gesetz. (Der Unterschied zwischen "absolut genauer" Wägung und "wesentlich ungenauer" Wägung ist ein grammatischer und bezieht sich auf zwei verschiedene Bedeutungen des Ausdrucks "Ergebnis der Wägung".) (*BT*, p. 140)

Wittgenstein contrasts two concepts of *weight of a body*. Both deal in a particular way with the results of measurement. The first does not leave any margin of tolerance. Thus, if the same body is measured within a few minutes with the results (75.1 kg; 75.2 kg; 75.1 kg), then, according to this concept, one needs to say that this body has gained and lost 100 g in those few minutes (even with no apparent cause). If, on the other hand,

the concept leaves a margin of tolerance on its measurements, of say 0.2 kg, then the fluctuations of the results do not imply, according to the second concept, that any weight has been lost or gained.

The distinction between "absolutely exact weight" and "essentially inexact weight" might suggest that a concept of weight whose results of measurement have the form "G ± g" is vague for this reason. But this would misunderstand the role of the margin of tolerance. For in this case, the limits of the margin are non-vaguely given. Consider the following result of measurement: "Roger weighs 75.1 kg ± 0.2 kg." This result can be transformed to an unspecific but non-vague range: "Roger weighs between 74.9 and 75.3 kg." That is, the question whether measurements have a margin of tolerance has no direct import on the vagueness of a concept.

Finally, Wittgenstein conceives of two ways to make the concept *heap* non-vague. His attempt is peculiar in two ways. First, he mentions "form and consistency" as things that would need to be specified (*BT*, p. 140). Simply counting the number of grains would not be sufficient for a definition of "heap", since the grains could be spread evenly over a surface. Secondly, he does not define *heap* in terms of numbers of grains but in terms of volume in m^3. This may have pragmatic advantages, since it is easier to measure the volume of a body instead of counting grains. However, it leads also to some peculiarities. Wittgenstein conceives of two ways to define "heap" in contradistinction to "heaplet".

a1 A collection of grains (of a certain form and consistency) is a heap iff $V \geq K\ m^3$.
a2 A collection of grains (...) is a heaplet iff $V < K\ m^3$.
b1 A collection of grains (...) is a heap iff $V > K\ m^3$.
b2 A collection of grains (...) is a heaplet iff $V \leq K\ m^3$.

On the definitions a1 and a2, it does not make sense to speak of a largest heaplet, on the definitions b1 and b2, it does not make sense to speak of a smallest heap. The simple, mathematical reason (which Wittgenstein doesn't mention) is that for any (real) number n_i which is smaller than K one can find another number n_1, which is smaller than K and larger than n_i. Thus, a precisification of "heap" along these lines produces results that are similar to what one would expect of the unsharp concept *heap*. There, it is to be expected that the order to pour the smallest heap does not make sense, since the order brings the boundaries into view, but unsharp boundaries do not have an exact location. This, however, does not mean that the expression "smallest heap" is devoid of sense. If the context indicates a comparison class which is manageable, one may declare that the

one who cuts the cake gets the smallest piece. Similarly, somebody could be assigned to disperse the smallest heap in a room.

It is noteworthy that these considerations about precise stipulations only apply if "heap" is defined in terms of volume, which is continuous, not in terms of numbers of grains, which are discrete. If we define "heap" as $N \geq K$ numbers of grains, then there is a smallest heap: it has K grains. The lesson that one can learn from these discussions is another one. It is possible to construct a sharp concept of "heap" which shares with the unsharp concept the characteristic that the order "make me the smallest heap that you'd still call by that name" is nonsensical and therefore impossible to carry out (*BT*, p. 141). However, there is one disanalogy: In the case of the artificial "heap", "heaplet" pair, it is determinate for every volume of the number n_i whether a collection of grains of that particular volume is a heap or a heaplet. There is no largest heaplet because of the involved infinity. In contrast, for the vague ordinary term "heap", it is *not* determinate for every volume or number of grains whether a collection of grains is a heap or not.

Conclusion

While neither the *Philosophical Remarks* nor the *Big Typeskript* directly address the sorites paradox, both discuss the topic of modern vagueness to some extent. The most striking difference between the two is that for the *Remarks*, vagueness, or blurredness, is a feature of experience, in a phenomenal understanding of "experience". The issue arises because the Wittgenstein of the *Remarks* holds that the language that refers to experiences is logically prior to a language that refers to physical objects; and he tries to establish the features of the realm of experiences. The blurredness of experience has two aspects: i) It is relative to concepts of exact measurement, which has its parallel in modern vagueness in the underlying dimension. ii) It is subject to higher-order vagueness (as modern vagueness).

In the *Big Typeksript*, Wittgenstein ceases to view vagueness purely as a phenomenon of experience, and talks, for instance, of the vagueness of commands such as "Make the board roughly 2 m long." He discusses the adverb "roughly", whose vagueness is hard to deny, devises a psychological experiment which demonstrates the vagueness of "seen length" (thereby providing the link to the *Remarks*), and creates a sharp version of "heap", which nevertheless shares with the ordinary concept the feature that the command "Make the smallest heap which you would call a heap" is devoid of sense.

6

Vagueness in the *Philosophical Investigations*

In the *Philosophical Investigations*, the word "vagueness" is mostly used in a group of paragraphs where *Tractarian* views are criticised. One of the main targets is the *Tractarian* demand for determinacy. This essay offers a reconstruction of Wittgenstein's critique of the *Tractatus* and of his views on vagueness in the *Investigations*.

A central difference between the *Tractatus* and the *Investigations* is that the former adheres to a referential conception of meaning, and the latter puts forth a rule-governed conception of language as an alternative. The discussion of vagueness brings forth an important qualification of the rule-governed conception of language: Even though language is rule-governed, it is not everywhere bounded by rules.

In the *Investigations*, the discussion of vagueness touches upon two different topics. It touches lightly upon vagueness in the contemporary philosophical sense, where the sorites paradox and gradual changes on a similarity dimension take centre stage. Wittgenstein uses the term "blurred boundaries" for what comes close to the modern sense of vagueness. Here, the contrast between the *Tractatus* and the *Investigations* is that the former claims that no concept of ordinary language has blurred boundaries, whereas the latter claims that many of them indeed have blurred boundaries.

The other topic is the *Tractarian* idea of a completely determinate language, namely that a sentence should have meaning no matter what the state of the world is (*TLP*, 2.0211; 2.0212). This implies that no statement made with a completely determinate language is indeterminate. This excludes the borderline indeterminacy of blurred boundaries. Conceiving language as rule-governed, the *Investigations* reacts first by bringing to attention abnormal cases where the writ of the rules for the application of words runs out. Second, the *Investigations* tries to show that the idea of a completely determinate

language does not make sense upon reflection. The argument to that effect considers the case of a potential misunderstanding or misapplication a rule. In that case, it seems that we need a new rule that regulates the application of the first rule. But since the problem recurs, an infinite regress looms, showing that no language could clear up all doubts, and would therefore be completely determinate. But this in turn gives rise to the skeptical challenge of the rule-following paradox. For it seems that the argument proves too much, and that its ultimate consequence is that no utterance has any meaning at all. This is where the discussions of understanding and following a rule come in, and where Wittgenstein offers a diagnostic solution of the skeptical challenge.

The question that links those two topics most clearly in the *Investigation's* discussion of the *Tractatus* is whether there could be a "vagueness in the rules" (*PI*, § 100). This expression, to put it concisely, means that there are cases for which the rules do not regulate whether a word can be correctly applied or not. In such a situation, we are at a loss for words. The latter point is of great importance for the contemporary discussion of vagueness. For it is part of all major ways of dealing with contemporary vagueness that there is no room for such an absence of regulatedness.[1]

The main group of paragraphs that is relevant for vagueness is §§ 65-115. However, the topic of vagueness is connected both to the differing conceptions of language of the *Tractatus* and the *Investigations*, and to the rule-following considerations. Therefore, it is conducive to include an overview of the beginning of the book (section 6.1). Another preliminary issue is the relation between family resemblance and contemporary vagueness (section 6.2). Those paragraphs (§§ 65-115) where vagueness is most explicitly discussed can be divided into two parts. The first formulates with Frege the logical problematicity of blurred boundaries (section 6.3). The second provides two responses to that charge: A pragmatic reminder that concepts with blurred boundaries are not useless, and a reminder that drawing a sharp boundary *changes* the concept (section 6.4). Moreover, Wittgenstein provides a number of examples of indeterminacy that differ in intriguing ways from modern vagueness, or even Alston's combinatory vagueness (section 6.5). The comparison of language with a game not only suggests that indeterminacy is due to the circumstance that certain cases lay outside of the scope of rules, but also invites an inherently dynamic conception of language (section 6.6). Furthermore, a vague language

[1] Raffman's new book *Unruly Words* contains some formulations that come close to this view. However, for Raffman, vagueness is not a case where the writ of the rules runs out, but where there are, so to speak, multiple sets of rules that completely regulate the application of words.

is often admonished for not being exact. Here, Wittgenstein's discussion of the relation of natural language to logic is relevant, in particular what role exactness has to play therein (section 6.7). In section (6.8) I draw a connection between Wittgenstein's rule-following considerations and reconstruct an argument that undermines the idea of a completely determinate language. Finally, in an extrapolation, I situate borderline cases in Wittgenstein's framework (section 6.9).

This essay attempts to elucidate the role of vagueness in the *Investigations*. Such an endeavour must be set in a broader interpretation of the work. However, I will not be able to discuss differing interpretations of the *Investigations*. Moreover, my organisation of the book into sections, and my emphases on certain paragraphs and topics do not aim to give the 'best' interpretation of the *Investigations*. Rather, these choices are conducive for an interpretation of it in the light of vagueness.

6.1. The beginning of the book (§§ 1–64)

Despite a large host of similarities between the *Tractatus* and the *Philosophical Investigations*, the latter breaks with the former and sets out to engage with the former's "grave mistakes" (*PI*, pref.). One great difference lies in their conception of language. For the *Tractatus*, language is well-ordered and sublime, whereas for the *Investigations* it is variegated and as messy as human life. Here are two passages – whose context I will not provide – that indicate the different *feel* of the books.

> Die Menschen haben immer geahnt, daß es ein Gebiet von Fragen geben müsse, deren Antworten – a priori – symmetrisch, und zu einem abgeschlossenen, regelmässigen Gebilde vereint liegen. (*TLP*, 5.4541)
>
> Wir werden sagen können: in der Sprache (8) haben wir verschiedene *Wortarten*. Denn die Funktion des Wortes "Platte" und des Wortes "Würfel" sind einander ähnlicher, als die von "Platte" und von "d". Wie wir aber die Worte nach Arten zusammenfassen, wird vom Zweck der Einteilung abhängen, – und von unserer Neigung. (*PI*, § 17)

Not only does the *Investigations* stress the variety of kinds of words, where the *Tractatus* distinguishes only between names and logical connectives. But it also states that the taxonomy of kinds of words is not set once and for all, but depends on the purposes and

whims of those who fashion the taxonomy. This is a far cry from the *Tractatus*, which equates an account of language with an account of logic and attempts to uncover the conditions for the possibility of language (see also § 97).

§ 1 starts with a quotation from Augustine. It serves two purposes. First, to illustrate a simple, but powerful picture: words name objects. This picture underlies, to begin with, a philosophical (referential) conception of meaning, which claims that "Every word has a meaning. This meaning is correlated with the word. It is the object for which the word stands." It also underlies the more complex meaning theory of the *Tractatus*. The second purpose is to intimate that the picture does not only enthral the author of the *Tractatus*, but also Augustine, whom Wittgenstein held in high esteem, and is therefore more widespread. A simple language game – the shopkeeper – serves to indicate in a very rough way three different kinds of words: sortals, numbers, and (non-sortal) predicates (to use a common taxonomy). This is the beginning of the alternative view of language that Wittgenstein proposes. Another central theme that is touched upon is that "explanations come to an end somewhere". This is also expressed in the idea that certain presuppositions have to be taken for granted in linguistic practice, for instance the propensities of a common human nature (§ 185, 206).

§§ 2–5 introduce another central topic of the *Investigations*: Methodology.[2] The referential picture of meaning (not the theory) alluded to above is true only of a narrow part of our language – the builders language game (§ 2). One method (cf. § 48) to make this apparent is to invent a language for which the referential conception of meaning is true. Shortly thereafter we get another piece of methodological advice:

> Es zerstreut den Nebel, wenn wir die Erscheinungen der Sprache an primitiven Arten ihrer Verwendung studieren, in denen man den Zweck und das Funktionieren der Wörter klar übersehen kann. (*PI*, § 5)

And this is what Wittgenstein is doing subsequently. In § 8 he enriches the language game of § 2 with elements of the shopkeeper scenario of § 1 (and the pair of deictic words "this" and "there"), and discusses and dismisses the claim that these words have reference. He goes on to review the variety of kinds of sentences (Sätze) or utterances (§ 18–24), and argues that ostensive explanations (in contrast to ostensive teaching) presuppose linguistic knowledge (§ 28–32).

[2]The §§ 89–133 include the highest density of methodological remarks of the whole book. I will ignore them here (I have written on them extensively in part I).

The passage from § 33 on aims more directly at the *Tractatus*. Ostensive explanation would be presuppositionless (in terms of linguistic capabilities) if there would be one single action of pointing to, for instance, the form of a thing, about which no doubts can arise.

> Weil wir nicht *eine* körperliche Handlung angeben können, die wir das Zeigen auf die Form (im Gegensatz z. B. zur Farbe) nennen, so sagen wir, es entspreche diesen Worten eine *geistige* Tätigkeit. (*PI*, § 36)

Evidently, not all cases of pointing to the form of an object are accompanied by the same bodily action. Here, Wittgenstein contends, a great temptation for theoreticians comes into play: Instead of a bodily action (*körperliche Handlung*), a *mental* action (*geistige Tätigkeit*) (which is always the same for all pointings to forms) is postulated. And if it is the mental act that individuates what is referred to, all cases of referring are cases where a mental act of pointing singles out a thing. This in turn suggests that "this" is the "*real* name" (§ 38).

Here, we are in the thick of the *Tractarian* theory of meaning.[3] This is no longer the simple view that the meaning of a *word* is the object it refers to. It takes on the form that meanings are objects and detaches the objects from words. It does so for the simple reason that what is referred to by a word can be destroyed, which would make the sentence of which it is part of senseless. Therefore, real names should refer to simple objects (§ 39, 44; see also chapter 4). In the following (§§ 46–64), the *Investigations* deconstructs the idea of simple elements, makes fun of the *Tractarian* notion of analysis, and engages with the idea that one cannot say of a simple element that it exists. This connects to what one might call the metaphysical underpinnings of the *Tractatus*, that cannot be stated, but only be shown, and thereby to one of its stranger, but also central themes. That a sound cannot have a colour was traced back to the fundamental nature of the simple objects. The later Wittgenstein compares this theme to paradigms such as the standard metre, and thereby tries to preserve as much truth of these ideas as possible. The destruction of a simple object can be compared to the destruction of a paradigm. For the *Tractatus*, in such a case the possibility of language is at stake. For the *Investigations*, the loss or destruction of a paradigm can lead to the loss of meaning

[3] The interpretation of the *Tractatus* that naming is a mental act is particularly disputed. See (Hacker 2001) for a defence. See also (*NB*, 16.6.15). The *Investigations* continues to elaborate on this topic in the so-called private language discussions, §§ 243–315.

of words (§ 55, 57). (The *Investigations* needs to be more guarded because not every change in meaning influences the identity of a concept. The replacement of the standard metre by a new specimen does not necessarily change our concept of metre. We may continue in our linguistic practice, and keep our folding rulers.)

We also receive a sketch of the *Investigations'* alternative to the referential conception of meaning. I have already stated that it claims that the kinds of words and kinds of sentences are variegated. Importantly, it connects language to human action.

> Ich werde auch das Ganze: der Sprache und der Tätigkeiten, mit denen sie verwoben ist, das "Sprachspiel" nennen. (*PI*, § 7)

Important is here that Wittgenstein conceives of language as something that is inseparable from what people do. His choice of the simple language game of § 2 underscores this idea: it is a game of orders, not of reports. The paradigmatic case of understanding an order consists in carrying it out. Thereby he differs in another way from the Augustinian picture of language, where understanding an expression would be to associate it with an object. The insistence on what people do with words is also apparent in the connection to use.

> Was *bezeichnen* nun die Worte dieser Sprache? [aus § 8] – Was sie bezeichnen, wie soll sich das zeigen, es sei denn in der Art ihres Gebrauchs? Und den haben wir ja beschrieben. (*PI*, § 10)

It is superfluous to ask for the referent of a word as a guide to its meaning, if we already know how to use the word. If it were otherwise, it would be dubious that we should understand the language games that Wittgenstein presents us with, and everything speaks in favour that we do. This is not necessarily to say that "meaning" and "use" are synonymous, but that to explain the meaning of a word is to say how to use it (see also § 43, 138). That is, the meaning of "meaning" is elucidated by *connecting* it with the meaning of "use". Finally, the analogy of speaking a language with playing a game (§ 7, 31) suggests that we can regard it as, or at least compare it with a rule-governed activity. To give the rules for a word would then be to explain its meaning. Rules can have various roles in a language game. They can be aids in learning the game, or tools of the game itself, or a hypothesis that describes the observed use of words satisfactorily, or what is answered to a query (§§ 53, 54, 82). However, here, a big caveat has to be made.

Justifications may run out without that a speaker acts in an unjustified way. That is, it is possible to make a justified move in a language game, even if there is no rule that is a ready candidate for justifying ones actions. This theme connects to the heart of the phenomenon of vagueness and will come to the fore in the following.

6.2. Family resemblance and sorites-susceptibility

In § 65, Wittgenstein addresses a complaint that the author of the *Tractatus* is likely to voice: By discussing various language games, the *Investigations* fail to tackle the central question: What is the general form of language? What is common to all languages? Against this, Wittgenstein asks "Must there be something in common?" and introduces the notion of family resemblance. He proposes that there are a number of concepts (sentence, language, game, number, etc.) that are applied to their instances not in virtue of sharing a set of properties. The instances of a general term F do not need to have a common feature *in virtue* of which they are called "F"; a common feature that would be both necessary and sufficient for the applicability of F. This has the corollary that there may be well-functioning general terms that cannot be defined in terms of individually necessary and jointly sufficient conditions.

The alternative that Wittgenstein proposes are family resemblance concepts.[4] The instances of such a general term, say G, are called "G" in virtue of similarities. Not all instances are called "G" in virtue of the same similarity, but there are rather a number or family of such similarities. These similarities, Wittgenstein proposes, criss-cross and overlap, thereby, as it were, forming a net. One observation that speaks for the existence of family resemblance terms is that definitions seem to play a rather subordinate role in the actual practice of language. When explaining for instance the notion of game, Wittgenstein's prime example of a family resemblance concept, a typical explanation would be to describe or show some examples of games, and then to add: "This *and similar things* are called 'games'" (*PI*, § 69).

The last bit brings the sorites paradox to mind (which Wittgenstein of course does not mention). All that is required for a sorites paradox for a concept F is an ordered similarity dimension δ. These green patches are similar in colour to those patches, this guy is similar to Warren Buffett in wealth (which can be quantified in terms of money),

[4] Wittgenstein does not use the expression "family resemblance concept", but speaks of "family resemblances".

this tool is similar to that hammer in its function. That, of course, does not imply that "green", "rich" or "hammer" are family resemblance terms. Furthermore, "green" is no family resemblance term because the few similarity dimensions that are associated to it apply to all instances of green, and do not overlap and criss-cross. Or, to put it conversely, all instances of green have a hue, a brightness, and a saturation (or can be presented as a mixture of a certain amount of cyan, magenta, yellow, and black). And "rich" may very well be amendable to a (vague) definition: a person who is rich has ownership of assets that grant her access to a comparably large amount goods or services through trade.

Are family resemblance terms sorites-susceptible? While I cannot completely dismiss the possibility, let me rehearse a line of reasoning that at least raises some doubt towards an affirmative answer. Now, the largest obstacle to coming to an unequivocal answer is that it is far from clear which concepts have a structure of criss-crossing and overlapping similarity dimensions, and which don't. The examples that Wittgenstein mentions are few and philosophically disputed, and, as far as I can tell, commentators have not seen it as one of their tasks to come up with more examples (cf. Bamborough 1986; Wennerberg 1998; Sluga 2006; Baker/Hacker 2005a; Ben-Yami 2016). At any rate, I will *not* be a laudable exception, and do not try to expand on the small stock of family resemblance terms (or at least candidates).[5]

Moreover, Wittgenstein's characterisation of family resemblance as criss-crossing and overlapping similarities is in need of elaboration. Ben-Yami (2016, p. 10) characterises family resemblance concepts as having "a plurality of criteria or of dimensions of resemblance that determine their application". According to that definition, "heap" would also be a family resemblance concept, because its application not only depends on the number of grains, but also on the distribution or form of the grains. Or for colour concepts the dimensions of hue, brightness, and saturation are relevant. Similar deliberations are available for other vague concepts. Accordingly, there would be no distinction between degree vague concepts and family resemblance concepts. Here is my attempt to come up with a fruitful distinction.

One-dimensional Vagueness: Cases of indeterminacy can be constructed by proceeding along one similarity dimension while holding other similarity dimensions constant.

[5]Here's a list of what I have encountered: art, chair, comparing, computing, deriving, game, language, law, number, poem, politics, reading, romanticism, science, sentence, understanding, and many psychological concepts. (Glock 1996a; Stevenson 1957; *PI*, § 164, 236).

Multi-dimensional Vagueness: Cases of indeterminacy can be constructed by combining several dimensions of similarity.

One-dimensional vagueness is sometimes called "degree vagueness", and multi-dimensional vagueness is based on what Alston calls "combinatory vagueness" (Alston 1964, p. 87). There are four combinations. Some concepts have neither vagueness, for instance the geometrical concept of *acute angle*. There might be concepts that have both sorts of vagueness, for instance "democracy". A society that holds elections every 1000 days is democratic, but what about a society that holds elections every 1001 days, etc.? But there can also be a combination of various factors that make it doubtful whether a society is democratic or not. The democracy index of the economist takes into consideration five features: electoral process and pluralism; civil liberties; the functioning of government; political participation; and political culture.[6] Then, there are those that exhibit only one-dimensional vagueness, among them the standard examples in the discussion of vagueness: "heap", "green", "bald". Even though more than one dimension of similarity is relevant for the applicability of these concepts, there are no indeterminacies or borderline cases due to a combination of their features. There are borderline cases for "bald" due insufficient number of hairs (sparse hair), or borderline cases due to uneven distribution of hairs (bald patches). But a person with a large enough round bald patch in the middle of the scalp is definitely bald, no matter how many hairs he has on the rest of his scalp. Finally, there are concepts which have only borderline cases of multi-dimensional vagueness. I will discuss some examples below.

A very important difference between one- and multi-dimensional vagueness is that only the former reliably gives rise to sorites paradoxes. The reason for this is that a sorites paradox requires a dimension of similarity where there are clear cases of F, clear cases of not-F, and borderline cases. One-dimensional vagueness guarantees by its definition such a set-up. But multi-dimensional vagueness does not. Indeed, often it is difficult to construct a sorites paradox for concepts that exhibit multi-dimensional vagueness. A complex index (such as the democracy index of the economist) that brings a variety of features into a single, gradual scale may or may not be amenable to a sorites paradox. In particular, the evaluations of the index (which were developed for the purpose of having a rough guide to the state of democracy in the world) may be amenable to criticisms such that two societies where one is clearly more democratic than the other may receive

[6] I am grateful to H. J. Glock for this example. See also http://www.economist.com/node/8908438.

the same value in the democracy index. Only an index free of such problems amounts to a similarity dimension with a continuous ordering.

Another requirement for a sorites paradox is that there is a specification of the similarity dimension such that an appealing Tolerance principle can be formulated. This requirement bedevils the proposal of Bueno and Colyvan (2012, p. 30). They claim that a sorites paradox does not require a total ordering of the similarity dimension, and that a partial ordering suffices. In a totally ordered set, for every pair of objects, a_x and a_y, and the ordering relation R(x,y), either R(a_x,a_y) or R(a_y,a_x) holds. A partially ordered set allows for pairs of objects where neither R(a_x,a_y) nor R(a_y,a_x) holds. Bueno and Colyvan propose to produce from a partial ordering one (of many) total orderings by leaving out certain objects. In doing so, they can generate a totally ordered series for "religion": "Christianity, Buddhism, Brazilian soccer, English Rugby, Minor League Baseball, schoolyard play." But any attempt to formulate a Tolerance principle based on an index that considers the relevant features brings up the cases that have been left out. Consider "If a social system with a religion-index of *n* is a not religion, then a social system with a religion-index of *n+1* is not a religion." Such a principle does not command immediate assent because the religion-index is unknown. But by considering it, the partial ordering of the cases is revealed, or even the breakdown of a continuous ordering. Accordingly, there is no Tolerance principle for "religion" that has a the strong appeal that Tolerance principles have in general.

I do not have a characterisation of family resemblance concepts that marks it out as a distinct category in the current context of investigation. Thus, I will simply call those concepts "family resemblance concepts" that Wittgenstein and his commentators have called such, and which I have listed in footnote (5). It seems to me that these concepts exhibit multi-dimensional, but not one-dimensional vagueness. All that seems to be assembled along one similarity dimension are clear cases of F, and borderline cases, but no clear cases of not-F.

Here is an example of vagueness without sorites paradox. A shoot-out or a duel is clearly not a game. Paintball is a clear case of a game. There are certain similarities between a shoot-out and paintball, not least because the latter is modelled on the former. There are two groups, they shoot at each other, there are rules for what happens if a player is hit or "dead" in paintball, etc. One important difference is that a shoot-out uses weapons that are lethal and hurt, whereas in paintball, the "weapons" used emit colour balls (which may hurt only a little). Now, the obvious way to make paintball more

akin to a shoot-out is to tinker with the "weapons", and make them more weapon-like such that a victim may not just be coloured after a hit, but it also may be seriously hurt, or killed. The question now is: Would such an activity be a perverted game, or no game at all? The considerations pull in both directions. One reason is that in normal games, people do not get killed as part of the goals of the game. So it shouldn't be a game. On the other hand, apart from the deadlyness of the bullets, the context of the 'game' remains the same: Two groups with no prior conflict meet, enter an arena and then shoot with real weapons on each other (and if there are any survivors, they might shake hands again). This would probably come with a very different outlook on life than our own. But why should such a society be unthinkable? The Romans also had what we might want to call perverted gladiatorial games. I take this to be a reason that supports that this activity is a game, perhaps a perverted game. What differs between this borderline case of a perverted game or non-game and clear cases of non-games such as a shoot-out or a duel is that the latter accord a very different role to the death of players. It is taken seriously, even though it is regarded as a necessary risk in a challenge for goods with an independent value outside of the context of the duel. Now, if it is true that the perverted game (or non-game) I have described is a borderline case for game, it seems that no change on the deadliness axis will change this status. What is missing in order to arrive at clear cases of non-games are other features (or similarity dimensions), such as the challenge for honour or power.

Compare this with the standard examples of one-dimensional vagueness, "tall", "heap", "red". The only thing that needs to change in a tall person for her to be not tall, or a borderline case of tall is her height. Her relative shape, and the proportions of her body, and everything else may remain the same. The only change needed for heap is the amount of grains. Again, the relative shape, the material, the consistency may remain the same. The only change needed for red is the hue, while brightness and saturation may remain the same. In all cases, non-tall persons, non-heap things, and non-red patches are reached simply by changing one similarity dimension. In the case of games, this is not the case. Change in more than one similarity dimension is required to reach a clear case of an activity that is not a game. But if so, there is no Tolerance principle along the lines of "if the bullets are a bit more hurtful" that would generate a sorites paradox.

Another example of multi-dimensional vagueness is "ripe blackberry". Several similarity dimensions are relevant (size, taste, colour, shape, texture, internal structure). The addition of "ripe" is necessary, since blackberries grow and have a variety of colours

(white, green, red, then blue) and tastes in their growth. Now, if one simply changes one of these features, one will not have a berry that is not a ripe blackberry, but a borderline case of it. A red blackberry that tastes like a ripe blackberry should, and has the right shape, etc., is a borderline case of a ripe blackberry. A blackberry that is blue, and has all the other features, but tastes like strawberry is a borderline case for ripe blackberry (though a blackberry that is sour instead of sweet is just not ripe).

These results have an important corollary. "Having borderline cases" and "being sorites-susceptible" are not co-extensional. There are concepts which have borderline cases but which are not sorites-susceptible, such as "game" or "ripe blackberry". Often, these concepts are being seen as co-extensional or two sides of the same medal (Smith 2008, p. 1–2). Now, my claim is not the same as to say that incomplete definitions lead to borderline cases, and that therefore borderline case and sorites-susceptibility part ways (see Fine 1975). If we say that numbers smaller than seven are nice, and numbers larger than seventeen are not-nice, we have said nothing about the numbers from eight to sixteen. One can resist the claim that these are borderline cases for "nice number" on various grounds. One is that they do not show the characteristic sliding slope (eight is not nicer than nine). Another is that there are no reasons that pull into different directions. There simply are no reasons at all to decide whether eight is a nice number or not. The borderline case for game is different. If the weapons are less lethal, the game is less perverted (or more of a real game). In addition, the bloodiness of the activity is a reason to not count it as a game, but the friendliness of the contest is a reason to count it as a game. In order words, borderline case and sorites-susceptibility part ways even while being faithful to a traditional concept of borderline case.

Wittgenstein is concerned with concepts which are applied due to a family of resemblances. These concepts, it appears to me, exhibit multi-dimensional vagueness but not one-dimensional vagueness. Thereby, they are not guaranteed to be sorites-susceptible. This change of focus serves to explain in part why Wittgenstein is concerned with indeterminacy, but not with the sorites paradox.

6.3. The logical problem of blurred boundaries

After Wittgenstein introduces the idea of family resemblance, the voice of temptation raises an objection.[7] It seems that such concepts do not have fixed boundaries *("feste Grenzen")*. But if so, it seems that such concepts are logically problematic. This topic comes up in three passages.

> "Aber dann ist ja die Anwendung des Wortes nicht geregelt; das 'Spiel', welches wir mit ihm spielen, ist nicht geregelt." — Es ist nicht überall von Regeln begrenzt; aber es gibt ja auch keine Regel dafür z. B., wie hoch man im Tennis den Ball werfen darf, oder wie stark, aber Tennis ist doch ein Spiel und es hat auch Regeln. (*PI*, § 68)
>
> Man kann sagen, der Begriff 'Spiel' ist ein Begriff mit verschwommenen Rändern. – "Aber ist ein verschwommener Begriff überhaupt ein *Begriff*?" (*PI*, § 71)
>
> Hast du für solche Fälle Regeln bereit, – die sagen, ob man so etwas noch "Sessel" nennen darf? Aber gehen sie uns beim Gebrauch des Wortes "Sessel" ab; und sollen wir sagen, daß wir mit diesem Wort eigentlich keine Bedeutung verbinden, da wir nicht für alle Möglichkeiten seiner Anwendung mit Regeln ausgerüstet sind? (*PI*, § 80)

In these passages we find a variety of formulations of what the logical problem is. In my view, they amount to Frege's worry that there are cases for which the application of a concept is neither true nor false. If so, the Fregean voice in the *Investigations* contends, the concept at issue is not a concept at all. For Frege, that makes statements in which the concept occurs not truth-apt. The *Tractatus* even denied their having a sense for that reason. It is not specified in § 71 which of those consequences Wittgenstein has in mind, but it is clear that indeterminacy is thought to be a problem.

The first formulation (§ 68) is that the application of the word "game" is unregulated (*die Anwendung des Wortes ist nicht geregelt*). This can be read as that there are cases where the rules for "game" do not regulate whether the activity or thing in question is a

[7] Following Cavell (1962, p. 92), we can distinguish between a voice of temptation and a voice of correctness. The voice of temptation brings to discussion philosophical difficulties that Wittgenstein himself has had; these are acts of confession. The voice of correctness tests these philosophical problems and tries to find a way out.

game or not. Therefore, this formulation is concerned with indeterminacy in general. The second formulation (§ 68) is that the language game is not everywhere bounded by rules (*nicht überall von Regeln begrenzt*). The example of tennis which immediately follows suggests that certain courses of action are not regulated at all (How high may one throw the ball?). In analogy, we may say of a concept that the eventuality of a recalcitrant world (as in the case of the chair) exceeds the scope of the rules, and that there are no particular rules that deal with such strange cases. Again, this is about indeterminacy in general. There is some allusion to a gradual transition in this paragraph, which could bring one-dimensional vagueness to mind: "What still counts as a game, and what no longer does?" (*PI*, § 68). However, as shown in section (6.2), the vagueness at issue here is multi-dimensional vagueness.

The second passage of § 71 makes a clear allusion to Frege, in particular to Frege's *Grundgesetze* (*GGA*, Vol. II, § 56). There, Frege compares a concept with an area. An area that has no clear boundaries is no area at all for Frege. Analogously, a concept that has blurred boundaries (*verschwommene Ränder*), that allows for borderline cases is no concept at all. As was shown in chapter (3), this use of "concept with blurred boundaries" is different from the modern use; it pertains not to indeterminacy in a transition zone, but to indeterminacy in general.

The third formulation in § 80 is that we are not equipped with rules for every possibility of the application of a word (*nicht ausgerüstet für alle Möglichkeiten der Anwendung eines Wortes*). This is basically what I have called "Propositional Determinacy" in chapter (4), and which I think is the *Tractarian* formulation of determinacy. In particular, the reference to "all possibilities" mirrors the formulation of the *Notebooks* which calls for a determinate truth-value no matter what is the case (*was immer auch der Fall ist*) (*NB*, p. 67). Again, this concerns a general kind of indeterminacy.

Therefore, Wittgenstein picks up and addresses the Fregean worry that indeterminacy in at least a single case leads to the loss of truth-aptness of a concept in all of its applications.

6.4. Two responses

There are two ways in which Wittgenstein reacts to this charge of indeterminacy. The first is to point out that indeterminate concepts *can* be used, the second is to claim that if sharp boundaries were drawn, if this indeterminacy was removed, such a sharpened

concept would not be the one that the speaker has used all along. Moreover, the second answer supports the first. If the concepts that are being put to good use in ordinary life indeed have blurred boundaries, it would be highly questionable to call them "useless". This is the problem of nihilism (see chapter 3).

The first strategy occurs in a number of different places. One has already been quoted above, the invocation of the game of tennis. It is a game, and is played, even though there are no explicit rules on how high a player is allowed to throw the ball. The game works even though there *are* things that are unregulated. And while some things are unregulated, others are regulated. Another similar response is this:

> Wie gesagt, wir können – für einen besonderen Zweck – eine Grenze ziehen. Machen wir dadurch den Begriff erst brauchbar? Durchaus nicht! Es sei denn, für diesen besondern Zweck. So wenig, wie der das Längenmaß '1 Schritt' brauchbar machte, der die Definition gab: 1 Schritt = 75 cm. (*PI*, § 69)

The measure of length 'one pace' is not exact. Different people have differently sized legs, and even for the same person, her paces will not always be of exactly the same length. But that doesn't mean that it is completely useless to say that "the mill is two thousand paces north". Equally, sharp boundaries could be drawn for "game", and therefore the borderline cases could be eradicated. But doing so will not prevent the vague concept from being useless. Nevertheless, Wittgenstein's example of "pace" differs from words such as "heap" in that the sharpening of measures of length has a clear pragmatic advantage, whereas the sharpening of "heap" does not. That, of course, does not make an unsharp "pace" useless, but makes its sharpened version *more useful*. (One might, however, argue that as far as *estimates* are concerned, an unsharp concept is just as useful as a sharp one, since the considerable margin of error of estimates masks the added precision, anyway.)

A third instance is already familiar from the *Big Typeskript*. "Stand roughly there" (§ 71, cf. chapter 5). The function of the adverb "roughly" is to make the request less exact. There is not a particular spot that the other should stand on. Of course, one could reply that it makes the request less specific, because a wider range of spots is acceptable, but that there is still a sharp boundary between the acceptable and the not acceptable spots. One can insist on that, but it is still unclear what the pragmatic value of such sharp boundaries is, since we can get by without them. Wittgenstein adds to this how such requests are typically explained.

> Aber ist es sinnlos zu sagen: "Halte dich ungefähr hier auf!"? Denk dir, ich stünde mit einem Andern auf einem Platz und sagte dies. Dabei werde ich nicht einmal irgend eine Grenze ziehen, sondern etwa mit der Hand eine zeigende Bewegung machen – als zeigte ich ihm einen bestimmten *Punkt*. (*PI*, § 71)

A typical explanation of such a request doesn't draw the boundaries within which the other is to stand. Rather, the speaker acts as if he marked out a particular point. But even that gesture does not demarcate a precisely circumscribed area.

The first response strategy strikes me as a particularly pragmatic one. The vagueness of a language or a word does not diminish its usability. And it is certainly one of the functions of language to be usable, to get things done, or as Austin famously said, to do things with words. How far can such an argument go? A philosopher deeply in thrall of an argument or picture will not be impressed. After all, Wittgenstein has not directly engaged with the reasons behind the demand for sharpness. Perhaps we can regard it as an initial wake up call. In terms of utility, a vague language is just fine. This implies that there are no *pragmatic* reasons for reforming our language by sharpening vague concepts.

The other response strategy appeals to the actual use of the word "game". Is it used with sharp boundaries, or with unsharp ones? § 68 voices the opinion that it is used with unsharp ones, and §§ 76 and 77 underscore this.

> Wenn einer eine scharfe Grenze zöge, so könnte ich sie nicht als die anerkennen, die ich auch schon immer ziehen wollte, oder im Geist gezogen habe. Denn ich wollte gar keine ziehen. Man kann dann sagen: sein Begriff ist nicht der gleiche wie der meine, aber ihm verwandt. (*PI*, § 76)

In § 68 Wittgenstein claims that *we* use the word "game" with unsharp boundaries. In contrast, in § 76, Wittgenstein speaks only of his *own* use. This passage chimes well with the methodological individualism that I propose (see chapter 2), and alludes to the theme that the rules that are in force for *my* concept, as *I* use it, cannot be completely unknown to me. A paragraph earlier, Wittgenstein asks:

> Ist nicht mein Wissen, mein Begriff vom Spiel, ganz in den Erklärungen ausgedrückt, die ich geben könnte? Nämlich darin, daß ich Beispiele von Spielen verschiedener Art beschreibe; zeige, wie man nach Analogie dieser auf

> alle möglichen Arten andere Spiele konstruieren kann; sage, daß ich das und das wohl nicht kaum mehr ein Spiel nennen würde; und dergleichen mehr. (*PI*, § 75)

If my concept of game is completely expressed in the rules that I could potentially give (and we might add: that I might potentially recognise, cf. Glock 2012, p. 66), then the logic and meaning of *my* concept could not be something which I am unable to recognise as my concept. If this is conceded, drawing a sharp boundary for a concept, which I see, upon reflection, as unsharp, achieves only one thing: It multiplies concepts. We have now two different concepts, a sharp one and an unsharp one. Then, "the affinity is just as undeniable as the difference" (§ 76). To claim that a speaker's concepts are sharp despite her inability to locate the sharp boundaries is to misdescribe her concepts. The purported rule "The smallest heap consists of 100 grains" is not a rule she is actually following. The effect of these two responses is to confront the Fregean charge with its nihilistic consequences. Given that our concepts are unsharp, and can fulfil their ordinary purposes, the move to abandon or reform them for logical reasons needs to explain how "logically defective" concepts can be useful.

6.5. Abnormal cases

How does indeterminacy fit into Wittgenstein's rule-governed conception of language that he proposes as an alternative to the referential conception of meaning and language? As early as in § 68 Wittgenstein claims that the concept of game is not "everywhere bounded by rules". This suggests that in the case of one-dimensional vagueness, there is no rule which would decide whether a borderline collection of grains is a heap. And in the case of multi-dimensional vagueness, there is no rule that would regulate all possible cases of combinations of the variety of similarity dimensions or criteria. On top of that, Wittgenstein considers a circumstance that reveals a different path to indeterminacy: abnormality. This category is perpendicular to the distinction between one- and multi-dimensional vagueness. Both kinds can lead to indeterminacy in normal cases, and in abnormal cases. Like other kinds of indeterminacy, abnormality clashes with the demand for determinacy that was endorsed by the author of the *Tractatus*:

Propositional Determinacy: A proposition is true or false (has a truth-value) in any possible state of the world.

The chair case is a counterexample to this demand for determinacy, because the situation is too abnormal. In one important use, the concepts of illusion and chair (or, if you like, material object) exclude each other and have clear application in simple cases. Consider a chair that is painted such that from one perspective, it fades into the background, thus creating the illusion that it has vanished. Or a projection of a chair makes us reach into empty air. But a chair that behaves like an ordinary chair for some time, vanishes completely, reappears, turns out to be not solid makes it doubtful what we should say. "There's the spooked chair again!" is perhaps the best we can do, but it is neither an illusion nor deserves the unmodified epithet "chair". Consider this:

1) There are three chairs in that room.

Is it false because one of the three is spooked? Or is it true because it is a chair, albeit a spooked one, after all? A Gricean might insist that it's true but misleading, (1) violates the maxims of quantity or relevance: The fact that one chair is spooked is a property important for anyone querying about the number of chairs available. The Gricean makes use of the new expression "spooked chair" (which receives its specific sense through the description above) and claims that spooked chairs are chairs, in contrast to painted chairs, which are not chairs. And this is a fine stipulation. But it would be equally justified to count spooked chairs not among the chairs; after all, they may dematerialise when you're about to sit down. As Austin in effect remarks about a related example, the fact that the current rules are not able to cope with extraordinary circumstances does not mean that new rules (or a longer description) cannot cope with them (cf. Austin 1961c, p. 68). While it's unclear whether the thing in question is a chair or not, it is clear that it is a spooked chair.

Wittgenstein's most explicit discussion of abnormal cases is the following.

> Nur in normalen Fällen ist der Gebrauch der Worte uns klar vorgezeichnet; wir wissen, haben keinen Zweifel, was wir in diesem oder jenem Fall zu sagen haben. *Je abnormaler der Fall, desto zweifelhafter wird es, was wir nun hier sagen sollen.* Und verhielten sich die Dinge ganz anders, als sie sich tatsächlich verhalten — gäbe es z. B. keinen charakteristischen Ausdruck des Schmerzes, der Furcht, der Freude; würde, was Regel ist, Ausnahme und was Ausnahme, zur Regel; oder würden beide zu Erscheinungen von ungefähr gleicher Häufigkeit — so verlören unsere normalen Sprachspiele

> damit ihren Witz. – Die Prozedur, ein Stück Käse auf die Waage zu legen und nach dem Ausschlag der Waage den Preis zu bestimmen, verlöre ihren Witz, wenn es häufiger vorkäme, daß solche Stücke ohne offenbare Ursache plötzlich anwüchsen, oder einschrumpfen. (*PI*, § 142, my emphasis)

While Wittgenstein does not say anything about the truth-value of statements about the problematic chair or about the new examples of fear and pain, he uses the phrase that in such cases it is doubtful what we should say. And in his discussion of the chair case, he implied that the extant rules do not to deal with such extraordinary cases. This, I believe, licences my interpretation that such statements as (1) are neither clearly true nor clearly false, and that this is so because such cases exceed the scope of the extant rules. This interpretation works best for the chair case. However, in other cases, Wittgenstein contends himself with making the more prudent claim that if certain normal circumstances were to be radically different, our language games would lose their point (*Witz*) or purpose (§ 345). The expanding and shrinking cheese is such a case. It would be pointless to sell cheese by its weight if it is to be expected that the weight changes without apparent cause. But that doesn't mean that "This lump of cheese weighs 1 kg" does not have a truth-value. There is no direct connection between a language game having no point and a statement having no truth-value.

The category of pointless language games is, I would say, not that far away from the Gricean category of conversationally inappropriate speech acts. One difference is that Wittgenstein does not say of individual speech acts (statements, questions, etc.) that they lose their point, but of whole language games. The language game of fear includes a whole range of potential speech acts that belong to the semantic network of fear talk: "You were very brave", "He's a coward", "Why were you afraid?", "Why do you shiver like this?". The last is of course a form of words that retains its point in a *different* language game (frost). We can use Gricean terminology to express Wittgenstein's point: If certain facts of nature were different, all speech acts of a certain area of language or a language game would be conversationally inappropriate.

However, other examples that Wittgenstein mentions without describing them in any detail lean more in the direction of indeterminacy (§§ 142). They exploit a logical feature of certain concepts for which it does not hold that " 'What sometimes happens might always happen' " (§ 345, see also § 344). Wittgenstein contends that even though sometimes people speak inwardly to themselves, it would make no sense to say of a tribe

that never talks outwardly that they do talk, but only inwardly. This is so because the basis for ascribing inward talk to a human being includes that on other occasions, he has talked in the normal way. If this analysis is true, the question can be raised how much normal talk we need to ascribe inward talk to a person. And Wittgenstein's contention here is that we cannot engage with that question without running up against indeterminacy at one or the other place. For some mixture of outward talk and silence, it becomes indeterminate whether we should ascribe *any* inward talk to a person. Clearly, having different warn calls for leopards and snakes would not be enough.

The chair case, in contrast, belongs to another kind of indeterminacy due to abnormality: sortals. Further examples can be found in Austin (a cat I have known for years delivers a philippic, a goldfinch suddenly explodes, quotes Mrs. Woolf), and in Waismann (a cat grows to enormous size, can be revived from death, a substance that satisfies all our tests for gold but emits a strange radiation) (Austin 1961c, p. 67; Austin 1961b, p. 88; Waismann 1945, pp. 121f). The last example is not very convincing since Waismann's description fits to isotopes of gold, and since the number of protons of an atom is constitutive for being gold. A better example would involve mixed results in the customary tests for the number of protons.

What these cases show is that degree vagueness is not the only source of indeterminacy. Alston's combinatory vagueness, and Wittgenstein's abnormal cases are sources of indeterminacy as well. Now, as far as I can see, they do not pose any new problems for the extant theories of one-dimensional vagueness. For one thing, the indeterminacy of abnormal cases is either one of one- or multi-dimensional vagueness (even though it leans towards the latter). For another, I think that the extant theories of one-dimensional vagueness can be extended to cover multi-dimensional vagueness without any major new problems. However, one might say that their extant problems are exacerbated; their extant absurdity increased. For epistemicism, God could not only tell which grain makes a heap, but also whether the spooked chair is a chair. For degree theory, the further dimensions lead to additional constraints for assigning acceptable truth-values such as 0.983465412 to statements. This makes the truth-value assignments more fine grained. More importantly, the abnormal cases suggest a reason for their indeterminacy which is in line with Wittgenstein's philosophy of language: The rules of a human language have grown into the role of giving guidance for normal cases; it is no wonder that they shall remain silent in abnormal cases.

6.6. Changing rules

The idea that the rules of language do not resolve borderline cases or abnormal situations can be made fruitful for a particular view of the *dynamic* aspect of language. The rules do not regulate the application of a concept F in all possible situations. Only what is likely to come up is regulated, and slightly unusual situations can be handled on the fly. Now, for Wittgenstein, the rules for the use are constitutive for a proper name (or a concept). Accordingly, changes of rules seem to threaten the identity of proper names – if the rules change, the proper name is a different one. However, for Wittgenstein, what we call "the same concept", "the same proper name", or "the same meaning" allows for changes in the rules. Moreover, these criteria of identity are themselves subject to vagueness. There are borderline cases for "same proper name".

Wittgenstein elucidates the dynamic aspect of concepts with a metaphor:

> Warum nennen wir etwas "Zahl"? Nun etwa, weil es eine – direkte Verwandschaft mit manchem hat, was man *bisher* Zahl genannt hat; und dadurch, kann man sagen, erhält es eine indirekte Verwandtschaft zu anderem, was wir auch *so* nennen. Und wir dehnen unseren Begriff der Zahl aus wie wir beim Spinnen eines Fadens Faser an Faser drehen. (*PI*, § 67, first emphasis mine)

This brings a temporal (or dynamic) dimension into the conception of language. The concept of number does not change merely because for instance real numbers are newly considered to be numbers. We might say a new fibre has been added to the old thread. Though, of course, it depends also on the purpose of the comparison whether it is still the same concept. But at least on one standard of "same concept", the concept of number has been the same since Eudoxos (or even earlier), and the addition of different sub-concepts, even complex numbers, has not changed the concept of number (cf. § 556). Yet, this is not, as Frege would say, a clearer grasp of the concept of number, nor an unravelling of a predetermined concept. Rather, on one standard of "same concept", the concept of number comes with an inherent openness. Such things we call "number", and those things too, and now there is no reason not to count complex numbers among numbers.

The clearest case of this dynamic aspect that Wittgenstein considers is the one of proper names in § 79. Under which circumstances would a speaker have reason to give up the following statement?

2) Moses did not exist.

Wittgenstein claims that a number of descriptions are connected to proper names such as "Moses" or "N". For evaluating the truth of (2), it matters whether parts of the biblical story are true or false. This resembles the cluster-descriptivist account of the semantic role or application conditions of proper names, which claims that it is a cluster of descriptions, namely a disjunction (Searle 1958, Strawson 1959, Cumming 2016). A problem of cluster-descriptivism is that a disjunction of descriptions is true even if only *one* of a great number of disjuncts is true. Consequently, such an account would have to claim that Moses existed even if the whole biblical story turned out to be false with the exception of one single part of the story. The next move of a cluster-descriptivist might be to propose that not *one* (or *all*), but simply *many* or *most* descriptions should be true. This seems to be a plausible move, though I think also that Goldfarb has a point when he says: "All that can be said generally about our practices is that if the need arises, there ordinarily would have to be some number of descriptions would bring in, and any, some, or most of them might be at stake depending on the particular circumstances. As a *theory*, of course, this is little more than: when we use a name, we ought to have something to say about it." (Goldfarb 1997, p. 186)

Moreover, Wittgenstein adds to these notes on the grammar or logic of proper names the mentioned dynamic aspect.

> Habe ich mich entschieden, wieviel sich als falsch erweisen muß, damit ich meinen Satz ["Moses hat nicht existiert"] als falsch aufgebe? Hat also der Name "Moses" für mich einen festen und eindeutig bestimmten Gebrauch in allen möglichen Fällen? – Ist es nicht so, daß ich sozusagen eine ganze Reihe von Stützen in Bereitschaft habe, und bereit bin, mich auf eine zu stützen, wenn mir die andere entzogen werden sollte, und umgekehrt? (*PI*, § 79)

For Wittgenstein, these props (*Stützen*) do not form one large disjunction; if too many descriptions (or props) are false, Moses did not exist; if enough are true, he did exist. But there is no clear threshold on how many need to be true or false. Moreover, the props that the speaker has available need not give a clear verdict in all possible occasions. What Wittgenstein proposes is that with a few props in hand, a speaker is able to deal with frequently occurring situations. His canvas is only partially painted. But if the need arises, he can add another brush stroke, and *decide* in such a case which of the descriptions are more important, or which similar description can replace an original one.

Wittgenstein returns to this idea in § 83.

> Und gibt es nicht auch den Fall, wo wir spielen und – 'make up the rules as we go along'? Ja auch den, in welchem wir sie abändern – as we go along? (*PI*, § 83)

There are two ways in which language is said to be potentially dynamic. First, by the impromptu introduction of new rules. If it turns out that in the right time period, a baby was taken out of the Nile by a Pharao's daughter, the speaker might insist that Moses did not exist on the grounds that this man did not lead the Israelites anywhere, or that he had nothing to do with the ten commandments. On the other hand, if it turns out that there indeed was a leader of the Israelites who gave them the ten commandments, the same speaker may without fault hold onto his denial of Moses' existence on the grounds that this man was not raised by a Pharao's daughter, and did not part the Red Sea.

Second, by the impromptu change of rules. That there was a man called "Moses" is one of many descriptions that govern the meaning (or application-conditions) of the proper name, and we may also call this a rule for the application of "Moses". What if the leader of the Israelites at the time of the exodus was called "Ramses", or "Sesom"? Clearly, both names bear some resemblance to "Moses". It is natural to adapt the rule to these variations of name such that a leader of the Israelites called "Sesom" would speak for the existence of the biblical Moses. Or if it turns out that the Israelites did not escape the Egyptians by crossing the Red Sea, but by incapacitating their horses and chariots through a ruse, this might first contradict the biblical description. But, in a second step, "Moses did exist" might then refer to a biblical-cum-secular figure, where the ruse speaks *for* the existence of Moses.

On this view, the proper name "Moses" is inherently dynamic. The partially painted canvas allows and invites further additions, without that the proper name would thereby be a different one. We are talking about the same Moses, even if his Red Sea adventure turns out to be a bitter lake adventure. There is no need for the speaker to have an antecedently fixed priority and weighting of all the descriptions that he regards as essential for the name. Rather, he can, to a certain extent without fault, adapt and change what he regards as essential for the applicability of the name. *Ad hoc* additions and changes to the canvas are integral to such a conception. This contrasts with a fixed and precise conception of language. On such a conception, a complex machinery with its weights and balances determines what is correct to say come what may. There is no room for improvisation (though we may trace through time changes of the complex machinery

itself). The real problem with such a view is (not elaborated on in the *Investigations*) that it ascribes rules to speakers that they do not *intend* to follow and do not regard as *reasons* that "Moses" has been correctly applied (or not).

6.7. Logic, exactness, and natural language

Vagueness is connected to logic, because logic assigns truth-values to statements, and because indeterminacy is a threat to Bivalence. A logical system such as Frege's first order logic sketches an ideal of a precise language. This then raises the question of the relation of such a logic with natural language. Wittgenstein discusses two ideas that he deems misleading: First, to regard logic as a norm or ideal against which natural language is to be measured. Second, logic as an underlying structure of natural language. The first idea is introduced thus:

> F. P. Ramsey hat einmal im Gespräch mit mir betont, die Logik sei eine 'normative Wissenschaft'. Genau welche Idee ihm dabei vorschwebte, weiß ich nicht; sie war aber zweifellos eng verbunden mit der, die mir est später aufgegangen ist: daß wir nämlich in der Philosophie den Gebrauch der Wörter oft mit Spielen, Kalkülen nach festen Regeln, *vergleichen*, aber nicht sagen können, wer die Sprache gebraucht, *müsse* ein solches Spiel spielen. (*PI*, § 81)

I find this passage quite puzzling, for the following reason: "Games" and "calculi with fixed rules" (*Kalküle nach festen Regeln*) are mentioned in the same breath. But on my reading, the primary target is the comparison of language with a calculus, such as the calculus of first-order logic, and not the comparison with games. The latter comparison has been used illuminatingly throughout the *Investigations*. Of course, that does not rule out that there are disanalogies between games and language, and there are (e. g. we cannot give up to use language as we can give up to play a game, cf. Ryle 1971, p. 364, see also Hanfling 2000, p. 54).

What is the role of the insertion "calculi with fixed rules"? It cannot be synonymous to "game", for this would contradict Wittgenstein's claim of its indefinabililty of § 65. If it is a disjunction, this would suggest that there is a major disanalogy between games and language, for instance that one cannot regard speaking a language as a rule-governed activity. However, limitations such as the idea that language is not circumscribed by rules on all sides apply to games, too, and are therefore not a disanalogy (tennis, § 68).

Therefore, I prefer to regard it as a specification. Accordingly, Wittgenstein claims here that the comparison of language with one kind of games, calculi with fixed rules, has severe limitations. The "fixed rules" of § 81 echoes the "fixed meaning" (*feste Bedeutung*) of § 79, and the context of proper names. There, as I have shown in the last section, the partial unregulatedness of proper names suggests that they are dynamic. Not every change in the rules amounts to a change in the identity of the proper name. In contrast, the rules of a logical calculus do not change over time; and if they did, it would be a different calculus. Moreover, a logical calculus nourishes the fiction that its applications are completely determinate no matter how the world is. Accordingly, Wittgenstein's claim here is that a natural language such as English can be compared with a logical system, but that the logical system is not a better version of natural language.

Nevertheless, if one regards a logical system as better than natural language, the following question suggests itself. Why can the ordinary concepts not be reformed in order to remedy their indeterminacy? Two answers can be found in the *Investigations*. First, how can our ordinary concepts do any work, given that they are indeterminate? This is supported by the two answer strategies of section (6.4). There, Wittgenstein argued that ordinary concepts *are* useful, and that drawing a sharp boundary would *change* the ordinary concepts. The challenge for a proponent of a reform of natural language then is to show that natural language has a philosophical defect, and still is useful. Nihilistic commitments, such as Frege's that most statements of natural language are not truth-apt are a serious drawback of such reform-friendly positions (cf. chapter 3).

Second, are there *philosophical* reasons for reforming language? The general line taken by Wittgenstein is to deny this, because the initial problem is thereby not solved, but sidestepped (§§ 132). However, this strategy depends on what we regard as the initial problem. Consider the sorites paradox. Someone might say the problem is that there is an argument with a paradoxical conclusion. We can solve the problem by claiming that the main premise, Tolerance, is false. This can be achieved by reforming language, by replacing the paradoxical ordinary concept *heap* by a reformed concept *heap**, where the smallest heap* contains exactly 100 grains. But, I object, this misrepresents the initial problem. For what gives rise to the paradox is that we want to know the truth about heaps. Is it true that one can always remove a grain from a heap, and it is still a heap? The reformist capitulates in face of the problem of the heap and instead solves the problem for heap*. In a philosophical investigation, it is no triviality to describe a problem in a way that reveals "our real need" (*unser eigentliches Bedürfnis*) (§ 108).

However, Frege's initial problem seems not to be the sorites but rather that a single case of indeterminacy threatens the truth-aptness of a concept. Without going into the details, one might say that this problem arises from Frege's theoretical concern with language and meaning, and his purpose to construct an artificial language for mathematical reasoning. But then, this problem is not the initial problem either. To find the beginning would be a research project in its own right.

The early Wittgenstein took over from Frege the demand for determinacy. His problem or problems can be associated with the question "What is it for a proposition to have a sense?". Thereby, the Fregean demand influenced his dealings with those problems and questions, in that this logical ideal was thought to be hidden beneath the surface of ordinary language. This then is the second idea on the primacy of logic, that the underlying structure of language has the form of a logical system. It is discussed in §§ 89–115, which has the character of a confession. It reveals intimate details of the intellectual life of the author of the *Tractatus*. The emphasis is less on the theories themselves, but on the psychological atmosphere surrounding and the pictures influencing those theories. For instance, the author of the *Tractatus* regarded a sentence as something remarkable and weird (§§ 93–95), and regarded logic as a pure crystal (§ 97). According to this picture, logic presents the a priori order of the world.

> Diese Ordnung aber, scheint es, muß *höchst einfach* sein. Sie is *vor* aller Erfahrung; muß sich durch die ganze Erfahrung hindurchziehen; ihr selbst darf keine erfahrungsmäßige Trübe oder Unsicherheit anhaften. – Sie muß vielmehr vom reinsten Kristall sein. (*PI*, § 97)

The main aspect of this picture is the independence of logic from any "empirical murkiness or uncertainty" (*erfahrungsmäßige Trübe oder Unsicherheit*). While the logic of the *Tractatus* does have presuppositions – the substance of the world – these presuppositions are not stained by the dust of empirical things, but present the preconditions for thought about anything empirical. This creates a contrast between the actual signs used for communication, empirical and murky, and the logical order that is to be found despite, within these signs. The signs have to be purified from empirical dust (cf. § 94).

This picture lies behind the demand for determinacy (§ 99; *TLP* 3.23), and behind the tendency to regard the logic of a sentence as something deeply hidden, and licences the idea that language has an underlying structure of the form of a logical system. The point of Wittgenstein's confession is to get a clear view on the pictures that subtly influenced

the theories of the *Tractatus*. Wittgenstein goes on to compare and contrast his earlier views with his current views. Thereby, he expounds the position of the *Tractatus* (and of the *Investigations*) in broad strokes. The *Tractatus* does not propose an ideal language, instead it contends that the ideal is to be found in ordinary sentences by analysis. "There must be perfect order even in the vaguest sentence" (§ 98). Moreover, two methodological points come to the fore that both bear on exactness. The first concerns the goal of doing philosophy.

> Wir beseitigen Mißverständnisse, indem wir unsern Ausdruck exakter machen: aber es kann nun so scheinen, als ob wir einem bestimmten Zustand, der vollkommenen Exaktheit, zustreben; und als wäre das das eigentliche Ziel unserer Untersuchung. (*PI*, § 91)

The author of the *Tractatus*, Wittgenstein confesses, was aiming for a state of complete exactness. This state would be achieved once a complete analysis of, say, a sentence would have been given. Later, in § 133, he contrasts this aim with the aim of complete clarity, which is achieved once the particular philosophical problem that bites us has been dissolved.

The second methodological point concerns the diagnosis of the source of many of the misunderstandings of the *Tractatus*. This source is a picture (§ 115), and it concerns the ideal of a completely exact language.

> Eine Vagheit in der Logik – wollen wir sagen – kann es nicht geben. Wir leben nun in der Idee: das Ideal *'müsse'* sich in der Realität finden. Während man noch nicht sieht, *wie* es sich darin findet, und nicht das Wesen dieses "muß" versteht. (*PI*, § 101)

Wittgenstein regards the idea of a completely exact language as a powerful one, that held his earlier self in thrall. Once the search for an exact language is started, it is difficult to step out of such a project. Stubbornly inexact parts of language are not seen as counterexamples, but only as temporary setbacks that require further research.

The most powerfully argumentative point that Wittgenstein makes can be found before his confession begins. We could say: The idea that logic has to have a primary role (whether as ideal or hidden structure) when conceiving of a natural language is supported by the exemplary exactness of logic. But why has language to be exact?

> Wenn ich Einem sage "Halte dich ungefähr hier auf?" – kann denn diese Erklärung nicht vollkommen funktionieren? Und kann jede andere nicht auch versagen?
>
> "Aber ist die Erklärung nicht doch unexakt?" – Doch; warum soll man sie nicht "unexakt" nennen? Verstehen wir aber nur, was "unexakt" bedeutet! Denn es bedeutet nun nicht "unbrauchbar". (*PI*, § 88)

This brings up the pragmatic point that I have mentioned earlier (section 6.4). To be inexact or vague is not to be unusable. But then, what is it to denounce something as inexact?

> "Unexakt", das ist eigentlich ein Tadel, und "exakt" ein Lob. Und das heißt doch: das Unexakte erreicht sein Ziel nicht so vollkommen, wie das Exaktere. Da kommt es also auf das an, was wir "Ziel" nennen. (*PI*, § 88)

Exactness is good, inexactness is bad. This explains in part the appeal that an exact logic has in thinking about language. What is ignored in this, according to Wittgenstein, is that where exactness is good, it is so because it reaches its goal better than inexactness. And this not only makes exactness relative to a goal, but also to the standard of exactness. To give the distance of a person to the sun with a margin of error of one metre is exact, but to order a table with the same margin of error is inexact. Here, the standard of exactness is different. Consider another example. The higher the number of digits known in the expansion of π, the more exact any computation that uses π. However, for computing the fundamental constants of the universe, an expansion of thirty-two digits is used.[8] Relative to that goal, more exact knowledge of π is idle, and given the limited computing powers of computers perhaps even harmful.

Wittgenstein's animadversions against a context- and purposeless exactness should not be conflated with criticism of logic. The problem is not logic itself, but the demands of exactness that are usually associated with it. For instance, in a model of an exact logic, the relation between an object and an extension is always a determinate one, whether it is either yes-or-no in classical logic, or a determinate degree in degree theory. In contrast, a vague logic which genuinely leaves certain things open is not prone to the charge of a purposeless over-precision.

[8] http://blogs.scientificamerican.com/observations/how-much-pi-do-you-need/

6.8. On the idea of a completely determinate language

The idea that a single case of indeterminacy makes the respective concept logically deficient requires that there could be a completely determinate language with no cases of indeterminacy. In the parlance of rules we might say that the rules of such a language are prepared for every possible situation, or that it is "everywhere bounded by rules" (§ 68). Against the idea of a completely determinate language two considerations can be advanced, one Wittgensteinian, one Wittgenstein's. First, why should a language be prepared to deal with all conceivable circumstances? No one can can foresee all conceivable circumstances. And the rules are either generated by human beings or by blind historical forces – why should this creation process guarantee a determinate answer in all cases? And if the other replies that the rules do not need to mention all conceivable cases in order to be completely determinate, one can point to examples of indeterminacy such as the spooked chair (§ 80). The second is the following argument.

> Ich sagte von der Anwendung eines Wortes: sie sei nicht überall von Regeln begrenzt. Aber wie schaut denn ein Spiel aus, das überall von Regeln begrenzt ist? dessen Regeln keinen Zweifel eindringen lassen; ihm alle Löcher stopfen. – Können wir uns nicht eine Regel denken, die die Anwendung der Regel regelt? Und einen Zweifel, den *jene* Regel behebt, – und so fort? (*PI*, § 84)

Does the idea of a completely determinate language make sense upon reflection? What would be a description of a completely regulated language game? Wittgenstein contends that there is none. The simple but powerful argument that he puts forth is that such a language would need to be able to remove all doubts, but that there is no rule that could achieve that. The trouble is that a rule needs to be applied in a certain way. This particular way may well be the most common and familiar to us, but that does not mean it is the only one. Doubts about the application of a rule are possible. In the next paragraph, Wittgenstein elucidates this by the example of a signpost.

> Eine Regel steht da, wie ein Wegweiser. – Läßt er keinen Zweifel offen über den Weg, den ich zu gehen habe? [...] Aber wo steht, in welchem Sinne ich ihm zu folgen habe; ob in der Richtung der Hand, oder (z. B.) in der entgegengesetzten? [...] Also kann ich sagen, der Wegweiser läßt doch einen Zweifel offen. Oder vielmehr: er läßt manchmal einen Zweifel offen, manchmal nicht. (*PI*, § 85)

How could anyone fail to follow a (well placed) signpost? Let's say that there are only three roads, and the road to the final destination has a signpost that points into the direction of the final destination. But this ignores that following signposts is a cultural technique, as is the construction and utilisation of roads. A person unacquainted with these techniques might very well be unable to follow signposts, and misinterpret them. Of course, this is only one example. Wittgenstein provides some others: tables such as a colour chart (§ 86), Moses again (§ 87), a cube (§ 139–41), and reading (§ 157, 160). The crucial point is that a sign does not force its application on its reader. Accordingly, doubts about the application of a sign are thinkable.

The argument could be paraphrased in the following way:

3) If there is such a thing as a completely determinate language, it allows no room for doubt.
4) Doubts about the application of any rule are possible.
5) These possible doubts never come to an end.

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6) Thus, not all possible doubts can be removed.
7) Therefore, there is no such thing as a completely determinate language.

While none of the premises and conclusions are mentioned explicitly in the *Investigations*, on my reading, §§ 84–88 are highly suggestive for all of them with one exception. For instance, (7) is suggested by reading of the following question as being rhetorical: "What does a game look like that is everywhere bounded by rules?" (§ 84). The exception is premise (3). It is only suggested by a wider reading of the passage. Nevertheless, there is a connection between indeterminacy and doubt. One of Wittgenstein's formulations of indeterminacy is that it is "doubtful what to say" (cf. § 142). Moreover, this is indeed what a completely determinate language should be able to achieve: clear up all possible doubts. Indeed, it is a premise that is likely to be endorsed by the proponents of determinacy with which Wittgenstein is engaged, Frege and the author of the *Tractatus*.

At first sight, it may seem that Wittgenstein undercuts premises (4) and (5). For one thing, he claims that the possibility of a doubt does not mean that we entertain it, or need to (§ 84). For another, he suggests that it would be better to say that a signpost, as a matter of experience, "sometimes leaves room for doubt, and sometimes not" (§ 85). But this does not mean that the premises are false; it is just that the motivation to raise those doubts is itself misguided. In that regard, the conditional structure of premise

(3) is crucial. Wittgenstein does not think that a language should be prepared for all doubts. But neither does he think that there is such a thing as a completely exact and determinate language.

Finally, from the claim that there cannot be a completely determinate language it does not follow that statements of actual languages are indeterminate. This has to do with the fact that the doubts that Wittgenstein raises would ultimately undermine any utterance having a determinate meaning, it would prove too much. That is why he is required to offer a solution to the skeptical challenge. In order to, on the one hand, use these skeptical doubts against the possibility of a completely determinate language, and on the other hand, retain the possibility of meaningful speech, Wittgenstein's solution should be what Kripke (1982, p. 66) calls a "skeptical" solution, and what Kusch (2006, p.,16) calls a "diagnostic" solution. That is, it does not offer the justification the skeptic seeks, but shows the skeptic's requirement for a justification to be misguided.

Wittgenstein's stance towards this issue is epitomised in the following passage.

> Der Wegweiser ist in Ordnung. – wenn er, unter normalen Verhältnissen, seinen Zweck erfüllt." (*PI*, § 87)

One could apply the thought about "normal circumstances" (*normale Verhältnisse*) to the question whether there is a determinate language. If one were to say that a language is determinate if it is able to deal with all doubts that are likely to arise, then a determinate language is indeed possible. This is the difference between a healthy skepticism, and philosophical skepticism. The solution to the threat of § 84 is parallel to the solution of the paradox of the so-called "rule-following considerations" in §§ 185–242.

Problems about following a rule come up in a slightly different form in § 201. Instead of a regress of doubts, we are confronted with the idea that any course of action can be brought into accord, or conflict with the rule. If so, the idea that there is a correct way to follow a rule would crumble. In my view, this fuses the threat of doubts of § 84 with the wayward pupil of § 185. There, Wittgenstein imagines a case of a teacher and a pupil, where the teacher has taught the pupil some basic operations with numbers, in particular how to add 2 to another number. In doing so, he has given the pupil all the explanations that he could give to himself. The pupil's understanding has been tried and tested up to 1000. Now he is ordered to continue the series beyond 1000, and he writes "1000, 1004, 1008, 1012". The teacher tries to correct him, but the pupil insists that he has continued *in the same way* as before.

In other words, there is a set of rules, which the teacher has taught the pupil, and which the pupil says he follows. Despite this, the pupil continues the series "+2" in a different way. The teacher can tackle the apparent misunderstanding of the pupil that something changes after 1000. But any additional explanations he gives are susceptible to the same sort of problem. What is it then that makes his way of continuing the series the correct one? The answer is that there was a problem in the challenge. The teacher looks bad because he is unable to provide further explanations to underpin his way of following the rules. But there is a notion of following a rule in a justified way which does not require him to provide further explanation (cf. McDowell 1984, p. 337). Whether somebody follows a particular rule R depends on his application of the rule. Whether somebody uses the word F in the same way as we, depends on his application of F. Whether somebody uses the same method of measurement as we, depends not only of his description of the method, but also on his results of measurement (cf. § 242). The pupil, after all, is *not* following the rule "+2"; if he follows any rule, he follows a *different* rule by virtue of the difference in application.[9]

The parallel between the paradox of § 85 that a signpost does not give conclusive guidance and the paradox of § 201 is that the excessive demands are based on a misunderstanding of a concept involved. In § 201, it is thought that to follow a rule in a justified way, a speaker has to provide a further rule. In § 85, it is thought that for speech to be well explained, the speaker needs to explain away all thinkable misunderstandings. But this ignores that it is the task of explanations to remove and prevent misunderstandings that are likely to come up, or that have actually occurred (§ 87). Accordingly, a speech act is well explained if it lives up to these demands. The signpost, together with the ordinary instructions, is well explained. In that way, the demands for an exact language are themselves misguided. Even worse, as shown by the argument above, it is not possible for a proponent of a completely exact language to satisfy his own demands. With that, a major motivation of the *Tractatus* for thinking that ordinary language is completely determinate falls away. Now the author of the *Tractatus* is finally able to regard language without the lens that uncannily rectifies its indeterminacies. Wittgenstein is now able to

[9] I should stress that I do not agree with the communitarian aspect of Kripke's and Kusch's solutions. The crucial point is that the applications that speaker B makes are a criterion for whether he has followed the rule R. If he continues in the wrong way, this is a criterion that he is *not* following rule R, *as speaker A understand it*. For this criterial aspect of the applications, it does not matter whether A's use agrees with that of the community. What is true, though, is that human beings generally agree on what it is to continue in the same way.

see the indeterminacies at the edges of language as what they are: cases where the writ of the rules runs out.

6.9. Borderline cases reconsidered

To finish, let me return to borderline cases and the idea that blurred boundaries of concepts do not impinge on the usefulness of those concepts. I have also said that in borderline cases, it is indeterminate whether the concept applies or not, and that the rules in force for that concept do not give any decisive ruling for these cases. There is no rule that would regulate the boundary. Calling to mind abnormal cases we may say that they are by definition characterised by their infrequency, or stronger, that they simply do not come up in normal circumstances. The same, though in a weaker form, I submit, can be said about borderline cases.

Neither the order for a chocolate muffin, nor the prayer for a blue sky, nor the warning from a dangerous dog is marred by the existence of possible borderline cases. Nor do borderline cases come up frequently in any of our ordinary talk. The boundaries of concepts do not come into view usually. However, there are two cases how they might come into view. The first is by a philosopher's attempt to find out more about the boundary. The second is if our concepts turn out to be too coarse for the pragmatic purposes of the speakers involved. For instance, if a tailor would use only main colour names such as "blue", "green", or "yellow", it wouldn't be too surprising that cases may come up where the tailor would be uncertain or inconsistent in his applications. Then, the order "Buy two bales of green cloth" may lead to real pragmatic troubles. The runner might buy cloth that he considers to be green, but nevertheless be reprimanded by the tailor that he has brought back the wrong colour! And neither tailor nor runner are objectively at fault (though the tailor is in a better position to push through his version), because the cloth the runner brought is sufficiently similar to green to be called "green", but also enough similar to blue to be called "blue" (see chapter 8).

One pragmatic way to deal with such problems is to introduce a colour system with more divisions. Instead of just "blue" and "green", the tailors may also have a colour called "turquoise", situated between them (or an even more fine grained colour chart). In the discussion of vagueness, the standard objection against such a move is that the boundaries between blue and turquoise are just as blurred as those between green and blue, and that thereby the sorites paradox is not solved, but multiplied. And I agree. But

the point of the tailors solution to their continued disputes about colour was not to solve the sorites paradox, but to put an end to their disputes. They did this by introducing new rules or concepts *if the boundary in fact comes into view.* If the boundary comes into view, it has a disturbing and paralysing effect. The solution to this then is to lay out the concepts in such a way that the boundary never comes into view.

Conclusion

Vagueness is brought up by Wittgenstein's proposal to regard the logic or grammar of "game" as a network of similarities, and the subsequent charge that such a family resemblance concept is lacking in sharp boundaries. The problem that Wittgenstein discusses is not that concepts with blurred boundaries lead to a sorites paradox. If my deliberations are correct, family resemblance concepts and other concepts with multiple relevant similarity dimensions need not be sorites-susceptible. They do not have a similarity dimension with a clear case of F, a borderline case, and a clear case of not-F. Such concepts, however, are logically suspicious for the *Tractatus* because they are not fully determinate. In Frege's words, blurred concepts are no concepts at all. And in the parlance of rules, such concepts are not everywhere bounded by rules. This leads Wittgenstein to consider a different but related case: abnormality. If the world is recalcitrant, it becomes questionable and indeterminate whether we are dealing with a chair, a cat, or a goldfinch.

Wittgenstein offers a number of considerations against the logical problematicity of concepts with blurred boundaries. First, the pragmatic admonition that such concepts are not useless, and, second, the reminder that if one draws a sharp boundary, one *changes* the concept – it is not the same that has been in use all along. If sharpness is an ideal, it is either something that our concepts strive to achieve or is already present but hidden in our concepts. But the question remains why the ideal is to be preferred. Here, Wittgenstein offers a third consideration that continues in the drift of the pragmatic admonition. The charge of inexactness is the reproach that something is less conducive to reaching its goal than its exact alternative. But this makes the value of exactness dependent on the goal of the enterprise – and it is doubtful that the demands for determinacy have any bearing on the actual use of language. Moreover, if one claims that the exactness is already present but hidden, one ends up postulating rules that the speaker could not accept as his own rules.

Wittgenstein's examples of abnormal cases that lead to indeterminacy do not pose new problems for the extant theories of vagueness, but they exacerbate existing ones. Moreover, their very abnormality chimes well with the view language should serve the purposes of its human speakers. Accordingly, it is imperative that the rules of language cover frequent and normal cases, but negligible if they do not cover abnormal and infrequent ones. Moreover, if a rule is lacking right now, nothing prevents the speakers to introduce new rules to regulate unclear cases. In doing so, they do not necessarily change the concept involved, for the identity of concepts may allow for certain changes. Further, the *Investigations* sports an argument that calls into question the very possibility of a completely determinate language. Such an idea makes demands of language with which it can impossibly comply. A language is completely determinate only if it is immune to doubt. But doubts about the applications of rules are always possible, and never come to an end. Finally, borderline cases of can be regarded as cases where the extant rules do not give a decisive ruling. But such cases do not come up often in a mature language. If borderline cases do come up, they could be avoided by devising new concepts and altering old ones.

My interpretation of the *Investigation* shows also that the topics that Wittgenstein touches upon can be expanded. In particular, the question why logico-philosophically deficient concepts should not be reformed merits further discussion. In addition, the Wittgensteinian phrase that a speaker should be able to recognise the rules that she follows requires elaboration. Moreover, the idea that a concept may remain the same even if some of its constitutive rules change would merit a discussion of its own. In what sense can we then say that the drawing of sharp boundaries *changes* a concept such as *heap*?

III

The Not-a-Rule Approach to the Sorites Paradox

7

Tolerance as a Rule

Clearly, there is a distinction between people who are slightly tipsy, drunk, and completely wasted. But how much and what kind of funny stuff does a person have to do in order to cross the boundary from tipsy to drunk? The matter, I take it, is a vague one. There is no last curve in the walk, no last slur in the talk that separates a tipsy person from a drunk one.

The sorites paradox exploits the lack of a clear boundary of vague concepts. This essay tries to provide a solution, which I call the not-a-rule approach, that draws on a Wittgensteinian framework that regards speaking a language as a rule-governed activity. It licences the idea that the Tolerance principle is not a rule, without that this would make its internal negation a rule, leading to sharp boundaries.

The task of this essay is to tackle the short sorites paradox that relies on the Tolerance principle. That, however, is not enough to completely address the challenge of the sorites. There is a second version of the paradox that proceeds with a large number of conditionals. For reasons of space, I have to defer its solution to another chapter (8). The grist of that solution is that the conditional sorites is a form of a forced-march sorites, that the forced-march sorites forces the answers of the speaker into a certain form, and that this form misrepresents the phenomenon at issue. And here, there is a parallel between the two pillars of my solution of the sorites, the not-a-rule approach. Since our concepts leave certain situations unregulated, one is bound to misrepresent them if one forces an answer. Just as Tolerance is not a rule, the totality of the conditionals is not a rule. How that squares with speakers' commitments to individual conditionals remains to be elucidated. For that purpose, I use speech act theory (see chapter 8). In contrast, the central contribution of this essay is to apply the concept of linguistic rules to the sorites

paradox. In that way, the not-a-rule approach will have two pillars: The solution of the short sorites (with an emphasis on rules), and the solution of the conditional sorites (with an emphasis on the force-march sorites and speech act theory).

I begin by reconstructing the paradox (section 7.1), and laying down the essentials of the not-a-rule approach (section 7.2). Next I explain what role rules play in my framework (section 7.3). Then I ask what the status of Tolerance is, and conclude that it should be a potential rule-formulation (section 7.4). This allows for the possibility that Tolerance is not a rule that we follow, and for this reason false. Moreover, there is a distinction between an internal and an external negation. To say that Tolerance is not a rule is to negate it externally. This does not imply that its internal negation is a rule (section 7.5). Finally, I consider whether ordinary speakers follow Tolerance, and what sorts of concepts people have that indeed follow Tolerance, or its internal negation, or related potential rules (section 7.6).

7.1. The sorites paradox

The sorites is a recalcitrant paradox, known since antiquity:

1) *Pointing to one grain of wheat:* This is not a heap.
2) If *n* grains are not a heap, then *n+1* grains are no heap, either. *(Tolerance)*

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3) *Pointing to a collection of ten thousand grains of wheat:* This is not a heap.

But with equal certainty as premise (1), we know:

4) *Pointing to a collection of ten thousand grains of wheat:* This *is* a heap.

Imagine a large bucket full of grains of wheat being poured on the ground. It seems absurd to single out one specific grain – launched from its vessel, flying shortly through the air, bouncing off other grains and finally coming to rest – a single grain that makes the difference. Even re-enacting the situation, as it were, in slow-motion by throwing the grains individually into a pile does not mark one particular grain as *the* grain. The boundaries for the application of the word "heap" seem to be blurred, no sharp cut-off presents itself. That is why the second premise (2) is so alluring. Nevertheless, it is the main suspect for leading into trouble.

Consider the other two claims, premise (1) and the negation of the conclusion (4). They are pretty innocuous, and eyebrows would be rightly raised if somebody were to go on and call a few disparate grains a "heap", or if another was puzzled by the command to relocate a heap when standing right in front of a truck load of grains shaped like a heap. Such antics would be reason to assume that something basic has gone wrong with these persons; be it that they suffer from illusions, have problems with their eyes, live in their own reality disconnected from others, or that they simply do not understand the word "heap" (without realising it).

What makes matters worse for finding fault with the innocuous premise and the negation of the conclusion is that there is a perfectly parallel argument that goes into the other direction. Instead of adding grains (the accumulative sorites), we might take grains away from what is clearly a heap. Accordingly, we may distinguish between an accumulative sorites and an subtractive sorites. One way to mark the difference is to employ different versions of the Tolerance principle.[1]

Accumulative Tolerance: If *n* grains are no heap, then *n+1* grains are no heap, either.
Subtractive Tolerance: If *n* grains are a heap, then *n–1* grains are heap, too.

In this way, the parallel argument proves that one grain *does make* a heap. Moreover, from the conjunction of (1), Accumulative Tolerance, and (4) both the negation of (1) and the negation of (4) follow. I will not go into the logical details, but one works using *modus ponens*, and the other *modus tollens*. Accordingly, denying premise (1) stops the accumulative sorites but does not avoid the reverse sorites. And vice versa for denying the first premise of the reverse sorites (4). Therefore, to deny only one of the innocuous claims is to accept one sorites argument (upward or downward). Now, while it is possible to avoid contradiction by carefully selecting which absurdity to embrace – that nothing is a heap, or that everything is a heap – the choice is completely arbitrary. At any rate, such a reformed use of the word "heap" will be as useful as a vendor's reassurance about each of his items that it is of "very good quality".

Then, the premise (1) and the negation of the conclusion (4) are beyond consideration unless one is ready to embrace nihilism about natural language. But then, there are basically two options to avert the paradox: I) to deny its validity, II) to deny premise (2), the Tolerance principle. Both options are *prima facie* not appealing.

[1] The Greek expression σωρίτης could be translated as "heaper" or "accumulator". Thus, etymologically, and only etymologically, "accumulative sorites" is redundant. See (Barnes 1982, p. 33).

Ad I: To deny the validity of the inference involved in the sorites paradox is *prima facie* not a very promising road. For one thing, the inferences involved in the short sorites paradox can be made surveyable by a long version, which uses a large number of conditional premises (see chapter 7). And the long version is valid in classical logic using the innocuous inference rule of *Modus Ponens*. And *Modus Ponens* features in countless other inferences that are unproblematic. For another, we know of closely related arguments that they *are* valid: First, there are chains of *Modus Ponens* arguments using vague concepts are valid. Since Smith is in London, he will attend the meeting. Since he attends the meeting, he will go for a drink. Since he will go for a drink, he will go out at night. Second, there are cases where a great number of small changes eventually do add up to a large change, without that there would be any doubt about the validity of the argument. Consider this situation. Berta has received a bundle of ten thousand rare old dollar bills. She makes it a habit to light one up and burn it after eating her breakfast egg. At first she doesn't notice any change to her stack of old dollar bills, but with the years, it becomes apparent that it dwindles. Yet, she still owns a stack of old dollar bills. However, after ten thousand lavish breakfasts, she has to accept the fact that her stack of old dollar bills is no more. It's gone. This wisdom is embodied in proverbs such as "Constant dripping wears away the stone" and "Little strokes fell big oaks".

Ad II: To deny the truth of the Tolerance seems initially more promising, since we have reason to be suspicious about it; it is a strong and general claim. It does not say of *some* crossings that they do not make a difference, but of *all* of them.[2] And here, one could think, the Tolerance principle overreaches. How can it claim that every one-grain-difference does not make a difference between the correct and the incorrect application of "heap"? When, obviously, there is a distinction between things that are heaps and things that are not heaps. That is one of the main functions of a predicative concept: Distinguish between things that fall under it, and things that do not fall under it. The sorites paradox threatens this distinction. And the Tolerance principle plays the main role in this, because it makes a sweeping claim about a large (and in our version unrestricted) number of one-grain-differences.

The road to go then is to repeal the paradox by repealing the Tolerance principle.

[2]This generality can be unrestricted, as in the English version above, or restricted, as in the case of the formal version with ten thousand conditionals. Unrestricted generality applies to an infinity of cases, whereas restricted generality states of a finite set that all its members have a certain property. Both kinds of generality produce the paradox, if the finite version reaches far enough.

However, to deny Tolerance, if it is a customary general claim, is to accept its negation. And if it is false that all one-grain-differences don't make a difference between heap and non-heap, then there must be a counterexample. One grain that makes the difference between there being a heap and there being merely a few grains laying around. The problem of this move is that it not only gets rid of the paradox, but also of the blurred boundaries of the concept of heap. There is after all, we are now committed to say, a sharp cut-off; only that we do not know (yet) where it is. However, by taking this route, we are encountering another problem just as severe as the vanishing of the boundaries necessary for categorisation. It is radically unclear what additional piece of information could cure our ignorance. Imagine an honest but perverted scientist that forces the choice on Clark between his life and providing the scientist with the smallest heap. Every sort of scientific instrument is at Clark's disposal. But which grain shall it be? Is there any reason to favour 113 over 114? Or should it rather be a matter of volume, or weight? And is the reason that Clark has strong enough so that the perverted scientist will have come to the same result? Note that the scientist's task would be less perverted if he merely slammed down a bible in front of Clark and challenged him to count the number of "e"s in it. For the latter is resolvable by known means, but the former is not. If there is a sharp boundary, it is radically unclear what sorts of considerations could uncover it. We have therefore good reason to assume that the issue of the smallest heap is irresolvable in principle, and not just as a matter of contingent fact.

Only a minority of the proposals for solving the paradox opt for option (I) and deny that sorites arguments are valid. Some proponents of many-valued logic (e. g. Machina 1976) claim that the sorites is invalid, but others take the line that it is valid with premises that are not entirely true (cf. Keefe 2000, p. 104). Another philosopher who claims that the sorites paradox is not valid is Ben-Yami. His proposal is of particular interest to me since he also draws on Wittgenstein to resolve the paradox. His proposal will be discussed in chapter (8).

Most of the main proposals for solving the paradox opt for option (II): They deny the truth of Tolerance. That means that they somehow have to make the acceptance of the (internal) negation of Tolerance acceptable.

Internal Negation of Tolerance: There is an n such that a collection of n grains is a heap and one of $n–1$ grains is not a heap.

A first option is epistemicism: It accepts that Tolerance is false, that there is a counter-

instance to Tolerance, and that the apparent absurdity of locating the boundary is that we do not know where it is. Of course, as shown above, epistemicism has to appeal to a special kind of ignorance. Williamson's proposal is ignorance about the boundaries of our concepts. For Williamson, the extensions of our concepts are determined by our use of them. Since a small change in use could easily go unnoticed, and since speakers don't know any algorithm to generate meaning from use, their opinions about the location of the boundary are not reliable, and hence have no knowledge about them. The task of the epistemic theory is then to elucidate how use could determine sharp boundaries and how speakers can follow a rule that they are unable to know.

Another way to deny Tolerance is to claim that it is false without there being any counterexamples. Normal general claims such as "All men are witty" are false only if there is counterexample, a man that is not witty. The Tolerance principle is a generalisation about one grain differences of the form "if 122 grains make a heap, then 123 make a heap". By allowing it to be false without there being one single grain that makes the difference between heap and non-heap, say the difference between 143 and 144 grains, a new possibility opens up. The falsity of the Tolerance principle would not imply sharp boundaries. Raffman's multi-range theory and supervaluationism regard the Tolerance principle as a general statement of that peculiar type. The task of such kinds of theories is then to elucidate what kind of general statement the Tolerance principle is supposed to be.

A third way to deny the truth of Tolerance is to say with the many-valued logic approach that it is not completely false but only false to a certain degree. Correspondingly, there is no sharp cut-off between true and false, but as the number of grains dwindle, the truth-value of "This is a heap" diminishes as well. The task of such a theory is then three-fold: i) Explain away over-precision: Is there a reason why the statement "This is a heap" has the truth-value 0.9987 instead of 0.9986? ii) How can there be degrees of truth if English has only two words for truth-values, "true" and "false"? What is the basis on which to ascribe such rules to speakers of English?

The core of the sorites paradox as I see it therefore consists in the challenge of reacting to the Tolerance principle. We cannot accept the truth of Tolerance on pain of being committed to its paradoxical consequences. However, denying its truth, both on ordinary reasoning and in classical logic, commits us to accepting its negation and the existence of a sharp boundary. But this leads to further hard questions: Why is the boundary where it is? And why do we not know (and are in no position to find out) where it is?

7.2. The not-a-rule approach

The present proposal, like many others, finds fault with the Tolerance principle. However, it is set in a framework that differs in its approach to language and the role of logic to all other approaches. What sets it apart is that it brings the notion of a rule to the discussion of the sorites. The core of this solution is to deny that Tolerance is a rule that speakers follow. Crucially, by not accepting Tolerance as a rule that is in force for our concept of heap, we are not committed to accept that the internal negation of Tolerance is a rule that is in force for our concept of heap.

This solution of the short sorites proceeds in three steps. It first raises the question concerning the status of Tolerance. Is it a rule the speaker follows, (or of the linguistic community of which he is a member)? Or is it an empirical statement? I claim it is not an empirical statement, and that its status should be that of a rule. Second, I lay out what the ramifications of that status are. One is that the real question regarding Tolerance is whether it is a rule that is in force. But then, how shall we think of P, if it is not a rule? It is crucial here that P's not being a rule is different from P being false *simpliciter*, which implies that not-p is true. For that purpose, I distinguish between an internal and an external negation. The internal one works as the customary negation. But the external one negates that it's a rule, and this can be made explicit by adding the that-clause "It is a rule (that is in force for subject S) that ..." to (2). Then, to evade the dilemma between paradox and sharp boundaries, Tolerance should be negated externally: Tolerance is not a rule that we follow. In other words, the *tertium non datur* doesn't hold for rules.

Tertium Datur for Rules: P is not a rule (that is in force for S) $\nvDash \neg$ P is a rule (that is in force for S).

The third step is then to show that Tolerance is indeed not a rule that we follow.

7.3. Rules in a Wittgensteinian framework

In my solution of the sorites paradox, I employ the concept of a rule, and of following a rule. I will engage in three ways with that concept. First, I elucidate the notion of following a rule within a network of semantic concepts that are connected to each other. Second, I relate the notion of a rule to Wittgenstein's concept of a language game. And

third, I tackle the question what considerations are relevant for judging that a speaker follows a particular rule.

To explain the notion of a rule, it is conducive to elucidate the whole network of the notions of meaning, understanding, use, application, explanation, and following a rule. Speakers may give explanations of the meaning of the words they use. These explanations may come in different forms, for example: i) By drawing connections between words: by paraphrasing "When I say that he parried your objection I mean that he refuted it by his clever answer", by using the material mode "Crooks are bad persons", or by using the formal mode " 'adroit' means very clever or skilful". ii) By giving positive examples "An example of a town that is in the countryside but not in the agglomeration of a large city would be . . . ", or by giving negative examples "That's a person whom I would call 'not nice' ". iii) By reference to a sample "Azure blue is *this* colour". iv) Proper names can be explained by pointing to the bearer of the name. Moreover, instead of saying that the speaker explains the meaning of his words we may also say that the speaker explains what he means, that he gives or states a rule-formulation or that he elucidates the meaning of his words.

If the speaker has given us an explanation of the meaning of his words, we may still ask whether this is in fact the rule that he is following. Usually, there is no doubt what explanations the speaker has given or what rule-formulations he has uttered, but it might be unclear whether they indeed express the rules that he is following. We have reason to doubt his explanation if it clashes with other explanations, or if it clashes with his applications of the word. For instance, if Dora explained to us that a screw is a kind of cookie, but then failed to provide any example of a screw-cookie, and continued to use screw in its ordinary sense of building material "I'm sorry, I don't have any screw of the right size", "Could you please go to the store and buy a pack of screws of the size . . . ?" and is then satisfied with what Ephraim brings, ordinary screws, we would have reason to believe that she is *not* following the rule that screws are cookies, though we should also inquire about her use of "cookie".

The question "What are the rules for 'screw' that Dora follows?" is equivalent to "What does Dora understand by 'screw' "? Of course, it would be helpful to ask her whether she uses the word ambiguously, and if so, to press her to explain more fully what at screw-cookie is. Just any cookie of the form of a screw? Or is it connected to a particular recipe? However, understanding differs from following a rule in that there is no equivalent for "Does Garret understand that in eight days the president of Moldavia

will come for a state visit?". Nevertheless, the connection between his understanding and his applications and explanations of the constituent words are similar to the connection between which rules he follows and his applications and explanations. The latter are criteria for saying that he has understood what has been said. If he gives contradictory explanations that would be a sign that he has not understood. For example, if he first explains what a state visit is by saying that the president of Moldavia will meet officials of the visited state, but then expresses his hope that the president will come to his home because a state visit consists of visiting the inhabitants of a state, he doesn't seem to have made up his mind what "state visit" means.

The questions "What does Dora understand by 'screw'?" and "What does Dora mean by 'screw'?" are equivalent. However, the question "What is the meaning of 'screw'?" is not equivalent, for where the former two were clearly related to Dora, the latter leaves open whether the meaning is in Dora's idiolect, in the dialect of some group, or, say, in British English. This last sense of the meaning of a linguistic community is also in play in Garret's example. Even if it turned out that he himself uses "state visit" consistently as a visit to the state's inhabitants, this would result in a misunderstanding if applied to a newspaper report in British English. Another particular use that the word "meaning" offers is to say that the sign "blubb" doesn't have a meaning. It wouldn't do to say that nobody understands it (nobody living understands Etruscan). Perhaps one could say that "blubb" cannot be understood, but then, we could give a meaning to it and then it could be understood. The closest equivalent, though perhaps a bit unwieldy, would be "There are no rules for 'blubb' ".

However, the difference between "What does Dora mean by 'screw'?" and "What is the meaning of 'screw'?" should not be exaggerated. Here, a number of Strawsonian distinctions are helpful (1950, pp. 325–9). First, Strawson distinguishes between sentences and expressions (parts of sentences). Second, he distinguishes between types, uses, and utterances (or tokens). For instance, the sentence-type "I'm sorry, all the screws are sold out" might be uttered by the shop-keeper on consecutive days. But if, unknown to him, a new shipment has arrived, his utterance is false on the second day, but true on the first. Moreover, if Ephraim, to make sure, asks not only the shop-keeper, but also the clerk, and they utter the same sentence, they have thereby made the *same* use of the sentence – their truth-conditions are the same.

Strawson is adamant that the categories of truth and reference apply only to uses of sentences, not to sentences themselves. The sentence is used to make a statement which

then can be true or false. Equally, the expression-type "the hardware store down the road" does not refer to anything, but if used in an utterance it may refer to a particular store. Moreover, he also insists that meaning (in one of its senses) applies to types, but not to uses. Now, in the sense in which Strawson explains what meaning is, this is certainly true: "To give the meaning of an expression (in the sense in which I am using the word) is to give *general directions* for its use" (1950, p. 327). We can also distinguish between general directions that explain how an expression is to be used in all contexts, and general directions for one particular context. In this way, we might say that "Cut the grass!" means that you should use the lawnmower and not that you should chop some bundle of grass into tiny bits. This gives general directions for "cut" because in the context of grass (or lawns) that is what you should do. In contrast, a particular direction is something that should be dismissed, after the particular situation for which it was created is over. This plays a role for vagueness, because, according to my view, the type-meaning of a vague word leaves certain things open, especially the boundary between where the concept applies and where it does not apply. In such cases, speakers can make *ad hoc rulings* to avoid occurrent borderline cases in their speech acts. That is, they can introduce a new rule, which is in line with the extant rules, that is dismissed as soon as the situation is over. This takes up the Lewisian idea of a conversational score that Shapiro applies to vagueness (Shapiro 2006, pp. 12f). If an *ad hoc ruling* sticks, and it changes the extant rules to such an extent that the concept is not the same, it introduces a new sense or context ("adult" in law), a new idiolectical or dialectical variant, or a new piece of jargon.

Following a rule differs from acting in accordance with a rule, in that one does not need to be aware of that rule, where in order to follow a rule, one needs to *intend* to follow that rule. For instance, employing a wide sense of "to act", we may say that the movements of the planets are in accordance with Newton's gravitation laws, but they do not follow these rules, since they don't have any intentions, and don't intend to do so. In contrast, players of chess follow its rules. Imagine that somebody not completely familiar with the rules of chess is surprised when a pawn is moved two squares as an initial move: "But that goes against the rules of chess". The players then explain to him that this is not so; that a priorly unmoved pawn may move to squares. These players clearly follow that rule because they can cite it to justify their actions, on top of their actions being in accordance with that rule. They allude to the rule as a reason for their action.

In that way, the player who moved the pawn two squares intended to follow that rule.

Of course, he did not recite the rule while moving the pawn, nor did he think of it. But the fact that he is able and ready to use this rule as an explanation, as a reason for his action, the movement of the pawn, shows that he intends to follow that rule. On the reverse, the observer had used *his* understanding of the rules of chess to criticise a move. He then was corrected on the grounds that the customary rules of chess do allow that move. This of course need not be the end of the discussion, since he might appeal to a university variant of chess where pawns can only one square at a time.

To intend to follow a rule is a necessary condition for following a rule, but not a sufficient condition. It is necessary, because if the player moves the pawn by accident, he is thereby not following the rules of chess, even if it's his turn, and otherwise in accordance with the rules of chess. It is not sufficient, because the player may misapply a rule that he intends to follow. For instance, a player may know how the knight is allowed to move on the chess-board, but attempt to make a move which is not in accordance with the rules of chess (cf. Baker/Hacker 2009, pp. 135–140).

Wittgenstein coins the notion of a language game, and there are a couple of points I want to highlight concerning the notion of a rule. The concept of language game can be applied both the whole of language, or a significant part of it. A significant part the whole language game is one which could be played if all the other parts were not played. For instance, the language game of classifying mushrooms into different kinds, and of sorting them according to their edibility and toxicality could be played by a tribe that has no other language games. i) Speaking a language is part of a wider activity. The whole language is part of the wider activity of living one's life, or of a form of life. Equally, the (partial) language games are part of a wider activity. Classifying mushrooms is part of gathering (and presumably eating) mushrooms. Speakers of that tribe not only say "fly agaric", "chanterelle", or "portobello", but also *do* something with the mushrooms. They pluck them from the ground, put them into their basket, show them to each other, bring them home, clean and cook them, etc (*PI*, § 7, 23). ii) Making a move in a game (such as chess) corresponds to a speech act, that is, asking a question, making a prayer, describing an object according to measurements. Similarly, just as a sentence may consist of smaller units, words, a move in a game may consist of smaller activities, rolling dice and moving the stones (*PI*, § 22). iii) Rules are central both for games and language games. The rules of a game are constitutive for the game; the rules of word are constitutive for its meaning. Moreover, to explain a game is to explain its

rules; to explain what a heap is is to give the rules for the use of the word "heap".[3]

iv) Can the use of a word fail to be regulated for all cases? Can the rules leave certain things open? Why not? Tennis is a game and yet has no rule for how high the ball may be thrown (§ 68). In this case, this does not need to lead to cases where it is doubtful what we should say. Since there is no rule, we have no grounds to forbid a particularly strong player to throw the ball very high into the air. We'll just have to wait until it comes down. However, if there is a strong crosswind, the ball might never come back on the court. What shall the umpire say then? Is it a foul? But the player has not even hit the ball! Is it not a foul? But the player has brought no ball to the other side! Wittgenstein contends that our language, and we, the speakers, have rules ready for the normal cases, but that we have no rules for abnormal, recalcitrant cases. For instance, we have rules for distinguishing between real chairs and illusions that look (and feel) like a chair. But consider the following case: Albert sees a chair, but when he wants to move it to the table, he loses sight of the chair. A few moments later, he rediscovers the chair a bit to the side, and sits on it. But a few minutes later, he gets up, and the chair disappears again. What shall we say? Is it an illusion, or a real chair? (§ 80, 142) Accordingly, our language does not have rules ready for all cases; it does not for abnormal cases. But, of course, since a mature language has rules ready for the normal cases, there is no *pragmatic* disadvantage. Moreover, there's a reason why a language cannot have rules ready for all cases. Consider a signpost for a trail in the mountains (with no letters on it). It is quite clear in which direction I have to go. But it is equally clear that a misunderstanding is possible if we allow skeptical doubts; I could interpret the signpost the wrong way around, for instance. Every rule that we give, every explanation can be misunderstood. But if the rule that explains the rule needs itself an explanation, we end up with a regress. In that way, only a language that contains an infinity of rules could be completely bound by rules that resolve every misunderstanding (§§ 84, 85, 185f) (see also chapter 6).

This brings me to my last point. What considerations come into play when determining

[3]The third point is exegetically somewhat of a stretch. There is no clear pronouncement of such a comparison. Nevertheless, one worry that is discussed in the *Investigations* is that the application of a word is not everywhere regulated by rules (*PI*, § 68). This implies that the application of a word *is* regulated by rules, whether everywhere or only in some places. However, one could now say that not Wittgenstein, but his interlocutor who is worried is committed to the view that rules are constitutive for meaning. Nevertheless, § 560 draws a close connection between explanation and meaning, and the §§ 68, 71, and 75 draw a close connection between rules and explanations. Therefore, the above interpretation is not without support.

whether a rule is in force for a tribe or a person? A good starting point are the applications that the speaker makes. To that we can count any kind of speech act where the concept in question is used, with the exception of the next category. So whenever a speaker says, asks, requests, prays or insults by using that concept, we have an application that in principle counts towards which rules the speaker follows. There are, however, exceptions to this. If the speaker makes an epistemic mistake, calls a cat a rat due to bad lighting conditions and speed, the application should neither count towards her concept of rat nor that of cat. Second, when interacting with the speaker, we will not only witness her applications, but also a whole host of normative interventions. I use "normative intervention" as an umbrella term for explanations, justifications, criticisms of applications of other speakers, and the reasons she gives for her applications. Third, speaking a language is embedded in the wider context of the other actions of the speaker, and they should be taken into consideration, too. If a speaker claims that dogs are dangerous, but shows no signs of prudence when dealing with dogs, we have reason to doubt that he really believes that. Similar considerations come into play when ascribing rules to speakers. For instance, a notorious skeptic might claim that there is no such thing as knowledge, but show no signs of uncertainty when going to his favourite restaurant.

7.4. What is the status of Tolerance?

Equipped with this notion of rule, we can engage with the first issue, the status of Tolerance: Does Tolerance have the status of a rule or of an empirical statement? In my view, there is a clear difference between the Tolerance principle and the other two members of the paradoxical set of statements (at least in the version of the paradox as formulated above). Since the sentences of the paradox are usually given without much context, (1) and (4) can be interpreted in at least two ways. One could regard them straightforwardly as empirical statements. In the case of (1), a true one if there is no heap and a false one if there is a heap. On the other hand, one could also regard them as explanation through paradigmatic cases: If anything is a heap, *this* (pointing to the 10'000 grains agglomeration) is a heap.

But Tolerance is more important. Could it be meant as an empirical statement? What distinguishes empirical statements from expressions of rules is that the former are contingent and can be verified or falsified by empirical means, while the latter are necessary and give rise to purely conceptual considerations (such as the sorites paradox).

However, there is the added complication that the fact that Tolerance is not a rule that we follow is an empirical matter; when we consider our use, we find that this is not our rule. The necessity comes only into play in a second step. If a sentence has the status of a rule, it expresses necessities. Whether it has that status, is an empirical matter.

Consider a paradigmatic empirical statement:

5) All heaps in this room contain exactly 155 grains.

It is quite clear by what method (5) could be tested empirically: By counting the grains of every heap in the room. Moreover, the statement (5) can turn out to be true or false. If, after counting, we conclude that it is true, the employee might pop a grain into his mouth, and, behold, (5) would be false. In contrast, as we have seen above, there are no known means in order to test Tolerance empirically. And it seems that we expect the answer to be necessarily true, and in this case general without exception. If Tolerance is indeed true, it would be inconceivable that it turns out to be false for one particular, recalcitrant heap, where we find out for this heap, and this heap only, that the boundary is at 142 grains. Of course, "heap" needs to be adjusted to context. If Tolerance is false, the boundary for a heap of spice and for a heap of peanuts are not going to be at the same place. Nevertheless, once adjusted for context, the answer is expected to be completely general and necessary.

Another option would be to regard Tolerance as a statement of causal regularity. Add one grain, and the agglomeration still is no heap. In that way, it could be similar to:

6) Add a crate to your lorry, and the bridge will still hold.

Again, causal statements are contingent, and the world could be otherwise. And there is a way to test causal statements empirically. Clearly, it is a contingent matter whether the bridge holds once the lorry advances. And if it indeed breaks, (6) turns out to be false. While the drivers might not have the means and the inclination to truly test (6), and even if there is only one such bridge, there are roundabout ways to corroborate such a statement. Build similar bridges, add crates one by one and measure how much the bridge budges, make educated guesses based on knowledge about material and construction, etc. To try to understand Tolerance along these lines would require that there are similar considerations available. But again, it is hard to see what we are waiting when we subject the heap to rigorous testing. Such a scientist might write down in his notebook: "I have

removed another grain. All testing reveals that the thing is still a heap." And we would very much like to know what his tests are. Whether he is inclined to *call* it a heap?

Or perhaps one could construe the status of Tolerance (or its internal negation) along the lines of Kripke's and Putnam's real essentialism. In that way, Tolerance would be metaphysically necessary, but epistemologically a posteriori. Such a proposal checks the box of necessity, but it is still unclear in principle what sort of empirical evidence could corroborate either Tolerance or its internal negation. Moreover, there is the question to what extent the real essentialist analysis of natural kind terms such as "water" can be applied to artefact terms such as "heap". Nothing counts as water unless it has the composition of the initial sample. In contrast, the most important feature of "heap" seems to be its geometrical form, and the materials of which it is composed can vary greatly: heaps of manure, of garbage, of corpses, of clothes. Then, perhaps we should understand "heap" as: Nothing counts as a heap unless it has the same form of the initial sample. The problem then is what it means to have the same form as the initial sample. Clearly, it cannot be the exact same form, or the exact same form in different sizes. But which forms and sizes are acceptable is exactly the problem of vagueness. Similar problems occur for nailing down the real essence of "red". The initial sample gives no guidance regarding which bandwidth of wavelength should count as "red".

These considerations suggest that the status of Tolerance or its internal negation is neither that of an empirical statement, nor a statement of empirical regularity, nor of a Kripkean metaphysical necessity. This suggests that Tolerance has the status of a rule-formulation. I do not want to rule out that further statuses could be identified, especially if one allows nuanced distinctions in the methods of verification, and different accounts of necessity. But I do not think that they would be a better fit for Tolerance while being a better account of necessity at the same time. To show this would go beyond the scope of this essay. My temporary conclusion then is this: Tolerance has the status of a rule. Here's why this status fits it well.[4]

Rule-formulations express necessities, can be tested by reflecting on the meaning of its constituent terms alone, and, if in force for a group of speakers, must be reflected in the reasons that the speakers can use to explain, correct, and justify the application of an expression. Regarding the last point, consequences of rules differ from rules, though they are as well necessary and true in virtue of meaning, and must be derivable from rules.

[4]For a defence of a Wittgensteinian conventionalist account of necessity, see (Glock 2003b). For a defence of analyticity along the lines of rules, see (Schroeder 2009).

Accordingly, analytic truths include not only rules, but also consequences of rules such as "In chess, in the position White: Kc3, Qa8, Be4; Black: Kar, Baz, White can mate in three moves" (Schroeder 2009, p. 104). Consider the rule-formulation:

7) Every rod has a length.

Every sign has a contingent relation to its meaning. The word "rod" could be used for trees. But, given that certain rules are in force, they are without opposite. What would it be for a rod to have no length, given how "rod" is usually understood in English? If something would be brought forth without a length, this having no length would be a reason not to call it a rod. Moreover, the truth of (7) can be ascertained by reflecting on the meaning of "rod", "length" and the other components of it. The things an ordinary speaker is prepared to call "rod" all have a length, and the absence of a length is a reason a speaker can cite for why a cube or a sphere are not rods. I propose to read Tolerance in this way. If it is true, it is so necessarily. Second, the way to find out its truth is to reflect on the meaning and use of "heap". Third, it should be a reason for the application of "heap" that speakers can upon reflection employ or accept.

7.5. Internal and external negation

The second step is to regard Tolerance as false not by applying an internal, but by applying an external negation to it. In that way, it does not follow that the internal negation of Tolerance is a rule; that there is a pair of collections of grains, n and n+1, such that one is a heap and the other is not. In that regard, "it is a rule that P (is in force for subject S)" is similar to "S has said that P" or "S believes that P". Consider the following case.

8) Kratylos has said that he was in the river.
9) Kratylos has *not* said that he was in the river.
10) Kratylos has said that he was *not* in the river.

If (8) is false, it follows that (9) true, but it doesn't follow that (10) is true. Why? Because Kratylos might have simply said nothing. Likewise, if it false that Ray believes that Moldovan is a football player, it doesn't follow that Ray believes that Moldovan is *not* a football player. Ray may have no idea who Moldovan is, or what a football player might be.

A disanalogy between rules, as I conceive them, and beliefs is that for the latter, the question of whether P is true or false is independent of the question whether S believes that P. P has a truth-value even if S neither believes that P nor believes that not-P. In contrast, if P is not a rule, then its status is probably (barring other options) that of an empirical statement. If it is denied as well that P has the status of an empirical statement (or any other status), what remains is a bare P. But then, such a bare P without any status is hard to make sense of and consequently doesn't have a truth-value. Indeed, it is usually quite clear what status any declarative speech act has even if it is not made explicit. "Peter has stolen my laptop. Those who steal are thieves. He's a thief." The first sentence can be empirically verified, the second lays down a rule, the third is empirical again. Moreover, for some declarative sentences it is neigh impossible to ascribe any status other than their standard status to them: One needs to do a lot of work in order to understand "There's no milk in the fridge" not as an empirical statement.

Nevertheless, some sentence-signs can be meant and understood as either an empirical statement or as a rule-formulation. If a thief is identified by cranial measurements, it makes sense to say: "That's a thief who has not stolen yet." Or consider the following sentence:

11) All acids (in solution) turn litmus paper red.

If (11) is understood once as a rule-formulation and once as an empirical statement, this has consequences for its meaning, and for the meaning of its parts. If (11) is a rule-formulation, then it defines what acids are. There simply could be no acid that does not turn the litmus paper red, exactly because that would be a reason not to count it as an acid. In contrast, if (11) is an empirical statement, then it can be falsified by substances for which we have an independent reason to call "acid" and that do not turn litmus paper red. In that case, (11) is not a rule for that speaker, and "acid" has a different meaning than in the former case.

What is the relation between the that-clause "It is a rule that ..." and P? There are two constraints arising from my position: It must make the external negation available and preserve the validity of the sorites. The latter is put in jeopardy by the following observation. We can abbreviate the standard formulation of the sorites paradox as "P, Tol $\vDash$ Q". For Q to follow from P and Tol, we need to apply *universal instantiation* to Tol. Then, in order to negate Tol externally, a that-clause is added to it: "P, R(Tol) $\vDash$ Q". But then, how can *universal instantiation* apply to R(Tol)? It seems that the latter formulation is valid only if "R(Tol) $\vDash$ Tol" is true. But then, Tol without any status or

that-clause should have a truth-value as well. One might then think that the not-a-rule approach is forced to claim that it lacks a truth-value.

Here are two complementary angles of response. For one thing, as I understand the that-clause, adding it to an ordinary language formulation of Tolerance merely makes explicit what is already there. It does not add anything new, but makes the meaning of Tol more surveyable. As above, it is hard to make sense of Tol that has not the status of a rule. Therefore, "P, Tol $\vDash$ Q" and "P, R(Tol) $\vDash$ Q" come to the same. Just as "Those who steal are thieves" and "It is a rule that those who steal are thieves" come to the same. For another, we may compare "It is a rule that ..." with "It is necessary that ...". What is the meaning and status of the second "P" in "$\Box$ P $\vDash$ P"? Bare P here has not the status of a contingent statement, since necessary and contingent statements exclude each other. No utterance of a speaker is to be interpreted as having the status of that bare P. Therefore, the bare P here is an auxiliary sign used to smooth logical transactions which cannot stand on its own. In that sense, the not-a-rule approach can accept "R(Tol) $\vDash$ Tol". For the latter has life only in logical book-keeping. My claim here is that the ordinary language formulations of Tolerance have the same meaning whether we add "It is a rule that ..." or not. Once we formalise them there may be pressure to always make the status explicit (also the status of empirical statements). In such cases, an auxiliary bare P might be helpful, but is without counterpart in ordinary language.

Another thing to note is that in certain circumstances for rules, the internal and the external negation do not differ. Consider

12) It is a rule of chess that pawns may *not* move two squares on the first move.
13) It is *not* a rule of chess that pawns may move two squares on the first move.

The effect of the two negations does not differ. In that version of chess, no two-square move is allowed for pawns. This is so because there is a further rule in the background to the effect that any move that is not allowed explicitly is prohibited. Nevertheless, for other rules, the negations differ.

14) It is a rule of chess that the rook does not cost 200 game-dollars.
15) It is not a rule of chess that the rook costs 200 game-dollars.

The difference is that (14) presupposes that chess pieces have an in-game cost, while (15) does not. Similarly, in the case of constitutive rules of language, there is often a difference.

16) It is a rule that crooks are not bad people.
17) It is not a rule that crooks are bad people.

(16) makes it impossible to find out empirically that some crooks are bad people, some good people. If (16) holds, it would be a contradiction to say "He is a crook, and a bad person". (17) allows for such an empirical discovery. Replacing "crooks" with "footballers" or "saints" makes the difference apparent. Saints should be, at least on one understanding, not bad people in virtue of the meaning of "saint". But the same is not true of "footballer". Some are good people, some are bad people. So an analogue of (16) should hold for saints, and an analogue of (17) should hold for footballers.

The application to blurred boundaries is the following. By applying the external negation to Tolerance, Tolerance can be false without that an internally negated Tolerance is true. In that way, the falsity of Tolerance does not imply sharp boundaries. Tolerance is false because it is not a rule that is in force for ordinary speakers of English.

7.6. Is Tolerance a rule that we follow?

The third step then is to show that neither Tolerance, nor a rule indicating a particular sharp boundary, nor internally negated Tolerance is a rule that we, the speakers of standard English, follow. What are the grounds to ascribe either of them to a speaker, say, Berta? In short, it is her applications of the concept, her normative interventions regarding it, and what role things that fall under that concept play in her life (see also section 7.3). I will concentrate below on statements and ignore other applications for simplicity's sake even though they are of equal importance.

I start by describing what I conceive as being the standard way of speaking English, of using vague English words, including *heap*, *green*, and *young*. This is case (i): Berta applies F to cases on one end of an underlying dimension δ, not-F to the other end, and in the middle ground she refuses to give a verdict in some cases, and for others, there is an overlap of her (fully contextualised) applications. She also gives paradigm cases for F and not-F, and mentions certain features in her explanations (for heap: form and size). *Prima facie* we can note here that first, her applications in the middle ground do not reveal a sharp boundary. Second, she does not volunteer Tolerance, its internal negation, or a rule indicating a particular sharp boundary. Such rules do not feature when she explains what a heap is, or justifies her applications, or criticises others. Then, we have no grounds to think that she follows either of those rules.

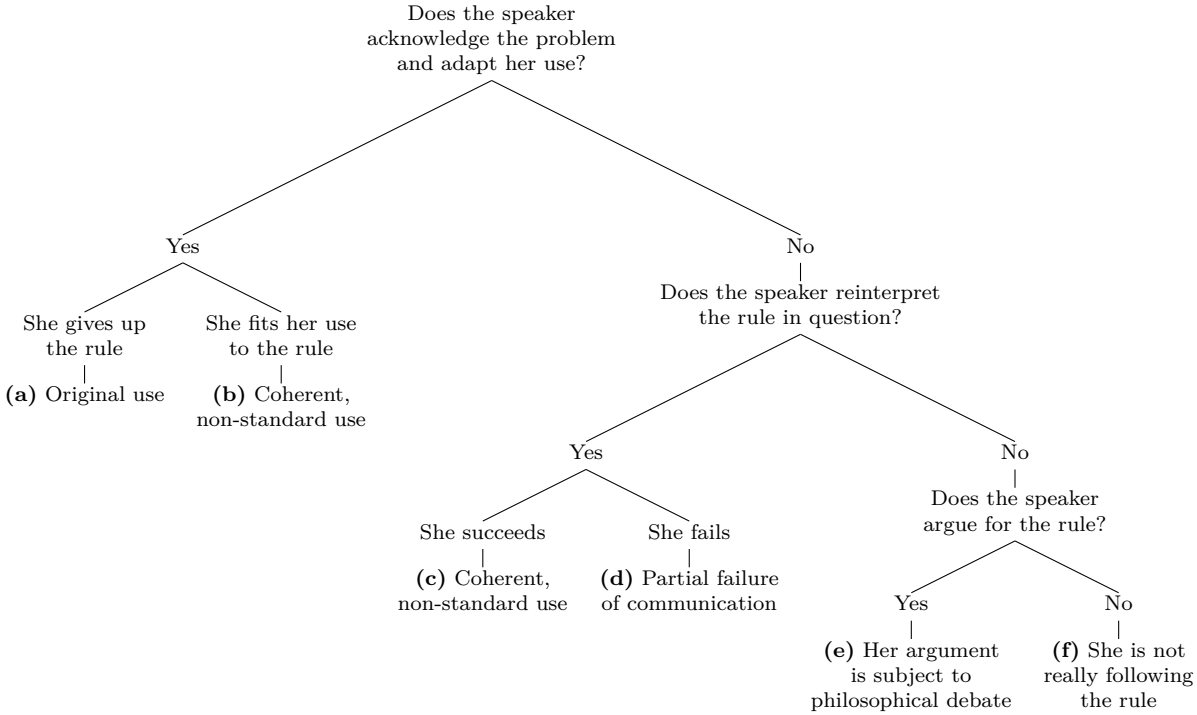
Whether this description of standard use is correct is of course an empirical question. A question about which I and you are in a good position to know since we participate in the practice as speakers of English (cf. Hanfling 2000, p. 54). Indeed, I think this *prima facie* issue is quite easily resolved, since one will find few speakers that volunteer a sentence such as (2) in all its mathematical glory. This is exacerbated by the ubiquity of vague concepts. For all of them, we would expect a speaker to volunteer a Tolerance principle, but we find so little.

However, it is not a criterion for a speaker following the rule R that the speaker volunteers R. It may suffice that the speaker assents to R. The real bone of contention then is the following. Speakers are liable to assent to Tolerance when confronted with the sorites. Does this not show that Tolerance is part of standard use? In my opinion, it does not. The reason for this is that the sorites paradox and the connected dilemma are an intellectual puzzle which is liable to lure the speaker into making an application or a normative intervention that is based on an epistemic mistake. And use that is based on an epistemic mistake should be discounted when considering which rules the speaker is following. Indeed, confronting a speaker with the sorites invites the whole philosophical reasoning that comes with it. And this involves both horns of the dilemma, sharp boundaries *and* the paradox, which is a genuinely hard nut to crack. If highly intelligent philosophers have spent decades to try to come up with a solution without coming to an agreement, one can hardly expect that a layman without logical or philosophical training should be able to withstand the lure of this puzzle. This argument does not depend on the intentions of the paradox-monger. A speaker unfamiliar with the sorites may wonder about the conceptual boundaries of "heap", and be drawn into the logical quagmire. Philosophers can be expected to have a solution to the sorites, but not ordinary speakers. That is why utterance in the context of the sorites cannot be taken as constitutive for the rules of the speaker. In that light, it would be highly interesting to have empirical evidence of actual speakers using vague concepts in *prima facie* non-standard ways, for instance volunteering Tolerance without the help of a sorites-schooled philosopher.

In order to address these issues in a fairly systematic way, I will consider a slightly different case, case (ii). It consists of the same use as (i) together with the speaker volunteering or assenting to Tolerance, say upward Tolerance. Because of the looming sorites paradox, this scenario raises the question how these two elements can be part of the same coherent use, how we can make sense of the behaviour of the speaker. Accordingly, we can regard it as an incomplete scenario, to be enriched by further interaction with a

speaker, say Berta. That interaction follows three topics. Does Berta acknowledge that there is a problem with her use, and does she adapt her use to avoid the problem? Does Berta have a non-standard understanding of Tolerance? Does she present an argument for Tolerance? I have arranged these topics in a diagram in figure (1). Note that this diagram is tailored towards scenario (ii), and that I want to highlight six scenarios that I find worth discussing. I start with the topic whether Berta acknowledges that there is a problem, and whether she adapts her use to deal with it.

Figure 1.: Interactions with a speaker with use (ii)



Assume that Berta acknowledges that there is a problem, and is willing to adjust her use to avoid it. Note that in this scenario, it is not quite clear why she volunteers Tolerance, or what the motivation of her interlocutor is who asks her about it, and gets her assent. The reason for this is that sorites reasoning is excluded by default in this scenario and relegated to case (e), because in such a case she would adopt Tolerance *based* on an argument. Nevertheless, faced with such a use, it is the duty of another interlocutor (us) to point out the contradictions she is committed to, and thereby invoke the sorites. So in this case, the sorites enters the discussion, but with the important caveat that Berta does not adopt Tolerance based on an argument.

There are two major ways for Berta to change her use. In the first case, (iia), Berta simply gives up Tolerance and reverts to the standard use (i). There is a strong resemblance to case (iie), where the introduction of arguments for or against a rule lead to a full-fledged philosophical discussion. Nevertheless, since the speaker has no proper motivation to utter Tolerance, and no argument for it, one might imagine that pointing out the ensuing problem might be enough to dissuade her from making that move. But it is certainly true that we are only one step away from a philosophical discussion of the sorites, and case (iie).

iib) A more interesting case includes that Berta adheres to Tolerance, and adjusts the rest of her use to avoid paradox. In that case, we indeed would have reason to say that Berta follows the rule of (say) upward Tolerance, but not the rule of downward Tolerance. She explains "heap" with reference to upward Tolerance, and she says of a few grains that they are not a heap. As a result, the upward sorites is triggered, and we get the conclusion that a truckload of grains is not a heap. Being aware of the problem, she now changes her use, and uses the paradox to justify how it should be changed. A coherent, non-standard use that she might come up with is the following: She never says of something that it is a heap, but in cases where there are some grains, whether only two or three, or a truckload, she sometimes says that there is no heap. She doesn't use "non-heap" (or "There is no heap") if no grains at all are present. Moreover, she explains the meaning of "heap" by upward Tolerance and gives us paradigm cases of non-heaps, namely a few grains (but not a lot of grains). She doesn't give us paradigm cases of heaps. If she would, she would contradict herself, because according to upward Tolerance, these are *not* heaps. Assume further that she never takes the observer to the side and whispers in his ear that she would never dare to call anything a heap because heaps are taboo. In such a case, we have reason to think that she follows the rule of upward Tolerance. The question now is what her words "heap" and "non-heap" mean, and what role they play in her life. Regarding "heap", we have reason to think that it has no meaning: She never applies it, *and* she doesn't give us any explanation for it. All we have are explanations for "non-heap" (or "not a heap"). "Non-heap" would therefore be a word without a positive form such as "inert" or "misgivings". So, what does it mean? I excluded above that Berta applies "non-heap" in any random situation, whether there are grains or not. If she would, her rules would licence her to apply it in *any* situation. But then, it does not draw any contrast anymore and cannot be used to say anything – one might then say that it would be meaningless, too. Now, because she

only applies "non-heap" if at least a few grains are around, we have reason to conclude that "That there is a non-heap" means that there are at least some grains around. In total, this is a case where Berta follows Tolerance, but has a very different concept of heap than the standard one.

Assume now that Berta sees no problem, and no reason to adapt her use. The next question is whether Berta has a non-standard understanding of Tolerance. Does she succeed in explaining a non-standard interpretation of Tolerance? Do we understand what she means? iic) Berta is able to explain and reinterpret Tolerance in a way that resolves the paradox. For instance, when we lead her through the paradox, she complains that we are misunderstanding Tolerance. She did not mean that we can add grains to a heap without bounds. Instead, she maintains that one grain doesn't make a heap, but that a buck load of grains does. Now, one might reply that it's hard to make out how she can defend her non-standard interpretation of (2) in this light. What is important here is to point out the possibility. If Berta succeeds to convince us that she has a coherent use, we can accept this as case (iic), a rational but non-standard use. Indeed, Ben-Yami (2010), proposes a solution of the sorites along these lines (see also section 8.6).

iid) Assume, in contrast, that Berta's explanations do not add up to a coherent or understandable explanation of her use. In such case, we have reason to assume that Berta suffers from a partial loss of rationality. Indeed, she interacts with heaps in the normal way, she is able to follow instructions, scoop up heaps, etc. It's only when it comes to Tolerance that we cannot really believe what she says, and it doesn't have a noticeable influence on her actions.

Assume now that Berta does not try to reinterpret Tolerance, and sees no problem with her use. The question then is whether she has a particular reason to volunteer Tolerance, to think that it is a rule for her concept of heap. For case (iie), assume she does provide an argument for Tolerance. In that case, our assessment depends on her argument. In the context of the sorites, we are likely to hear an argumentation that I have outlined in section (7.1). In particular, since Berta endorses Tolerance, it seems that she is particularly impressed by one half of the dilemma: In order to avoid sharp boundaries, one has to acknowledge Tolerance. Importantly, by arguing for Tolerance, her argument becomes subject to rational debate, and all philosophical moves become available. A first thing to point out to her is that Tolerance together with normal use leads to paradox. At this point, any philosophical solution of the paradox becomes available, and it is the outcome of that debate that will ultimately decide the issue. I,

of course, favour my own solution, which allows Berta to deny Tolerance without being committed to sharp boundaries. By showing Berta's argument to be flawed, we show that she has made an epistemic mistake, and we can discount her professed adherence to Tolerance for that reason. This is analogous to the case where we can discount for instance colour ascriptions under the influence of drugs, or in the case of an illusion. Making a distinction between *looking red* and *being red*, if the white cap of a bottle has a red spotlight on it, it is not red, but looks red. If Berta falls for the illusion, she might say that it *is* red. But that does not mean that her concept of red includes white objects. That particular utterance does not count towards her concept of being red, but should be discounted as an epistemic mistake.

iif) Berta again provides no reasons for Tolerance, does not reinterpret it, and claims that that is how she uses the concept of heap. But this time, she does not acknowledge that she runs into a problem with Tolerance. She does not change her behaviour, and goes on distinguishing heaps from non-heaps, just as in the normal case (i). We may compare this situation to reading a text, and encountering a contradiction. In the case of the text, we may, despite all attempts at charity, come to the conclusion that the author just contradicts himself. Similarly, we may conclude that Berta contradicts herself, that her use is incoherent. We would have to ascribe to her a partial failure of rationality. In the case (iid), this failure of rationality was quite isolated, because we just don't understand what Berta means by Tolerance, but can interact with her regarding heaps in the normal way. Equally, in this scenario, Berta does not draw the consequences of Tolerance, and is not aware of the problem. Because of that, the rest of her use is not affected, and her use in (iif) does not resemble the one of (iib) or (iic). Therefore, the failure of rationality is also quite isolated in (iif).

However, it is of course possible to construct a scenario (ii*) where Berta does not only volunteer Tolerance, but also where her use is is a mixture of the normal case (i) and the consequences of Tolerance. In that scenario, Berta would sometimes call a collection of 10'000 grains a heap, and sometimes she would say that it is not a heap, in a completely random pattern. This, I guess, would be a case of a more entrenched and widespread contradiction in her use. The question then is how isolated this contradiction is. Can we isolate it to heaps, does she only go crazy when talking about heaps? Or is it also applicable to all vague concepts? The more widespread the contradictions are, the less reason we have to think that she actually speaks a language. I mention scenario (ii*) because we need not restrict ourselves to scenarios with a very isolated contradiction,

such as (iif). However, I think that (iif) is more interesting because the incomplete scenario (ii) is very close to standard use (i). And as long as our goal is to understand standard use, the stranger scenarios only serve as a foil.[5]

Indeed, if we take this scenario a step further, we may further minimise her failure of rationality, in that she simply is not following a rule that she professes to follow. Consider an analogous case in non-linguistic behaviour. Berta professes to follow the rule "Do not eat for two hours before doing sport". Assume that you catch her violating that rule. If it is only in a few instances, or she can explain away the counterinstances as exceptions (a small salad does not count as a meal), or the rule is overridden by other considerations (her schedule forces her to eat at that point), we may still count her as following that rule. But if the divergence is systematic, we have to conclude that she does not walk the talk.

In the linguistic case, any rule could potentially be subject to the criticism that it is a rule the speaker professes to follow but does not. For instance, it could be the paradigmatic explanation to the effect that 10'000 grains are a heap. But then, Berta should not call large collections of grains a heap, which runs counter to our initial assumption that she uses heap in the standard way (i) (and volunteers Tolerance). In contrast, a scenario where Berta ceases to call large collections of grains "heap" is scenario (iib), where Berta ends up with a coherent, non-standard use. But in the current scenario (iif), the evidence suggests that Berta professes to follow Tolerance, but does not. The reason is again that the contradiction, if Berta does not change her standard use (i), is restricted to the utterance of Tolerance. Her failure of rationality is isolated to her utterances of Tolerance. We can ignore it, and everything else stays the same.

The moral of the story here is that if we start with the normal case (i) and add to this that Berta volunteers (or assents to) Tolerance (ii), we do not end up with a scenario where Berta uses "heap" or another vague term in the standard way. Instead we get a variety of cases where we may criticise the speaker's arguments, do not understand her explanation of what Tolerance means, have reasons to believe that she does not follow the rules that she professes to follow, or observe a difference to standard use. The scenario (ii) is not stable.

[5] I should add that my interest in standard use is as a means to an end. I conceive of the goal of philosophy as solving problems that I or my interlocutor have, or that are widespread in current society, or that are otherwise relevant. If my interlocutor speaks in an idiolect that diverges from standard use, that idiolect will have precedence. But philosophy is not a case of radical translation, and standard use is the proper starting place, especially in written philosophy with an unspecified audience.

Another feature of the discussion about which rules Berta follows in these different scenarios is that it cannot be separated neatly from philosophical issues. The interdependence between these scenarios and philosophical argumentation is foremost in case (iie), where an introduction of sorites reasoning and of epistemic mistakes bring the philosophical discussion of solutions to the sorites on the table. Crucially, which move counts as an epistemic mistake is itself subject to philosophical debate, as is our stance towards Tolerance. Similarly, in the case (iia), I claim that Berta might simply give up on Tolerance. This presupposes that denying Tolerance does not imply the acceptance of its internal negation. And for case (iif), the interpretation that Berta is not following the rule requires that utterances can have the status of a rule.

I will now discuss two cases analogous to (ii), where a rule is added to the standard use (i), a particular sharp boundary (iii), and internally negated Tolerance (iv). I will thereby concentrate on cases (b) and (f). I neglect cases (c) and (d), because I am not aware of any reinterpretation attempts of such rules. And I neglect cases (a) and (e), since these are just cases which invoke or are likely to invoke philosophical debate, which is roughly the same as in other cases. The core interest here then is to highlight the mismatch between standard use (i) and the particular rule, and show how the (rest of the) use could be adapted to the rule.

In case (iii), the speaker volunteers a rule that fixes a particular sharp boundary. Here, we don't have to choose between upward and downward Tolerance, because Berta can just postulate one sharp boundary between F and not-F. There are several ways to make "heap" sharp, and not all of them fit equally well into our lives. A first proposal is:

18) The highest point of a heap of wheat should be higher than this stick.

The advantage of that proposal is that this is a rule that Berta could actually follow. In close cases she might ask for a heap-stick, and if she cannot get one say that she doesn't know whether it is a heap. Another proposal is:

19) The smallest heap of wheat has 432 grains of wheat.

It thereby provides a rule which Berta can follow without expensive measurement equipment. The downside is that in many cases, she will have to resort to counting, of claiming ignorance, where the reason for her ignorance is that she does not know how many grains are present.

iiif) In the best case, the boundary is set in the middle ground (or it clashes with her clear cases of F or not-F). Even then, there are three instances of mismatch between the rule and the (rest of the) use. First, there should be no cases where she refuses to give a verdict *without* thereby referring to an epistemic problem. For instance that she doesn't know the number of grains, that she doesn't have the stick ready, that the heap is too widespread to apply the stick, etc. But in many cases of refusals, this is not what she would reply if we assume the standard use. Moreover, the formulations she might use in refusing a verdict don't *all* allow for an epistemic reading. She might say: "I can't say whether it's F or not", "I don't know whether it's F or not", "I don't know what to say", "It's kind of F, kind of not-F", "It's neither F nor not-F", "It's both F and not-F", etc. While some of these locutions are easily amenable to epistemic talk, others are not. Second, whenever there is an overlap in the middle ground, one of them must be an epistemic mistake of Berta, for instance in virtue of bad lighting conditions, miscounting the grains, or a wrong guess. But there is overlap without epistemic problems in the normal case. Third, in some cases, Berta *guesses* the numbers of grains, or the height of the heap, and is open to correction by more knowledgeable fellows. But that doesn't seem to happen in the normal practice. Again, if the mismatches are suitable isolated to the additional rule, we may conclude that Berta is not actually following that rule, even if she professes to follow it.

iiib) It is clear what has to change in Berta's use such that she can reasonably be said to follow one, and only one, of the two rules (18) or (19). If she refuses to give a verdict, her reasons for doing so are epistemic, and she can explain without further ado why she does not want to give a verdict "I don't have my heap-stick ready" or "I simply don't know how many grains there are, and it seems to be a close case." Moreover, disputes about whether it is a heap can easily be resolved by counting the grains, or applying the measuring stick, and Berta and her peers use these resources. Finally, sometimes, Berta guesses that an agglomeration of grains is a heap, but let's herself get corrected if someone has a credible claim to have counted the grains, or to have measured the height.

iv) There is a more roundabout and abstract way to adhere to sharp boundaries without following a rule such as (18) or (19). Berta might profess to follow the internal negation of Tolerance as a rule such that there is some grain that makes the difference between heap and non-heap, without specifying the location of the boundary (see also p. 138). As in case (iii), the boundary should better be located in the penumbra region, in order not to clash with clear cases on either end. One might say that the paradigmatic

explanations that Berta gives make sure that the boundary must be in the middle ground. So the resulting rule is that somewhere in the middle region there is a sharp boundary. On this scenario, there is one major mismatch between the volunteered rule and standard use (i). The mismatch concerns how Berta deals with borderline cases. On (iv), there is a correct answer to "Is this a heap?" even in borderline cases, though Berta does not need to know which is the correct answer. In such cases, she should refuse verdict with the reason that it is a case where she has no way of knowing on which side of the boundary it is. Cases of overlap then are cases where something has gone wrong epistemically. There is no leeway to just go with the flow. If Berta is ordered to scoop up the small heaps in a room where there is a progression of thirty closely resembling heaps, she should reject the order on the grounds that neither she, nor the one who has given the order have a way to know which are the small heaps. Indeed, we can expect Berta to cite the rule that somewhere there is a sharp boundary in dealings with borderline cases. In that way, the necessary adjustments for a coherent, non-standard use are clear: An awareness of the ignorance in borderline cases. That would be case (ivb). Conversely, if Berta only pays lip service to the internally negated Tolerance, and does not adjust her borderline case behaviour, we have reason to believe that she does not actually follow that professed rule (ivf). Of course, Berta could start to slap the sorites on her interlocutor. But then, we have a case of (ive), and her interlocutor can join the philosophical fray.

Conclusion

If I am right, the not-a-rule approach is able to escape both horns of the dilemma posed by the sorites paradox. It is not committed to the absurd consequences of the sorites argument, because it regards Tolerance as false. It is not committed to the absurd claim that our concept of heap has sharp boundaries, because it regards Tolerance as false not because there is a grain that makes the difference between heaps and non-heaps. But because Tolerance is not a rule that is in force for our concept of heap.

In a first step, I argued that Tolerance should have the status of a rule-formulation rather than that of an empirical statement, a statement of empirical regularity, or of a Kripkean metaphysical necessity. For one thing, the first two don't have the required necessity. For another, all of the other candidate statuses require that Tolerance can be verified or falsified empirically, but it is radically unclear what kind of evidence would do that.

In a second step, I clarified how Tolerance can be false without its internal negation being true. The key to this is to take its status of a potential rule or rule-formulation seriously. If it is purported to be a rule, we can falsify *that* by paying attention to the use of the relevant linguistic community or speaker, to their applications, explanations, criticisms, and justifications. Moreover, if it is falsified that P is a rule for S, this does not verify that not-P is a rule for S. In that way, the falsity of Tolerance does not entail the truth of its internal negation, and thus imply sharp boundaries. Its status as a purported rule allows for it to be externally negated.

Finally, I argued that if Tolerance, its internal negation, or a particular boundary were part of the rules of a linguistic community, the concepts of that community would be different from our ordinary concepts. They might have a concept that resembles our concept of heap, but differs in crucial aspects.

8

Borderline Cases in Speech

There is no rule which regulates the boundary of a vague concept. It is not a rule for "heap" that a collection of 41 grains is no heap but one with 42 grains is. Conversely, it is not a rule that you can always add a grain and it is still no heap. Accordingly, the Tolerance principle is not a rule that speakers follow. This is one pillar of the not-a-rule approach that I presented in chapter (7). The status of Tolerance is that of a rule, not of an empirical statement. As such, it can be negated externally; Tolerance is false because it is not a rule, and this does not imply sharp boundaries. In that way, the not-a-rule approach provides a simple solution to the following, short sorites paradox:

1) *Pointing to one grain of wheat:* This is not a heap.
2) If *n* grains are not a heap, then *n+1* grains are no heap, either. *(Tolerance)*

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3) *Pointing to a collection of ten thousand grains of wheat:* This is not a heap.

But with equal certainty as premise (1), we know:

4) *Pointing to a collection of ten thousand grains of wheat:* This *is* a heap.

This plausible argument is unsound because premise (2) seems to be true, but is false. This, however, is not the end to the troubles that the sorites puts us in. There is a version of the paradox that operates with 10'000 conditionals of the form "If 41 grains do not make a heap, then 42 grains do not make a heap." (see section 8.6). Indeed, the large number of conditionals that are employed in the conditional version of the paradox are implied by Tolerance. So far, denying that Tolerance is a rule just implies that not all of

the conditionals are rules. However, this alone does not amount to a satisfying solution of the sorites paradox for two reasons. First, singling out a number of conditionals in the middle that are not rules eliminates one sharp boundary between heaps and non-heaps at the expense of introducing two sharp boundaries between heaps, unregulated borderline cases, and non-heaps. This is the problem of higher-order vagueness, which is a classical problem for many proposed solutions to the sorites: The real problem is relegated to a further level. Second, unsharp boundaries imply that there are borderline cases. The issue then is to give an account of borderline cases that both evades the problem of higher-order vagueness, and of introducing a third truth-value through the back door.

In effect, these two challenges demand for a more systematic description of the features of the conceptual landscape that lead into paradox. For sure, this is a reasonable demand. However, it often pairs up with the forced-march sorites, which is a way of inquiring about our concepts that can lead to distortions in their description. My aim then is to meet the challenges of the conditional sorites and of an account of borderline cases that avoids introducing new sharp boundaries (higher-order vagueness), additional truth-values, and the distortions of the forced-march sorites.

An important intermediary step is to consider the speech acts at issue (e. g. "This is a heap" or "If 41 grains do not make a heap, then 42 grains do not make a heap.") in their natural environment. Such a natural environment can be characterised both negatively and positively. Negatively, it should not involve any abnormal behaviour of the natural world, or any cases of logical puzzlement (or other cases of illusions), where the speaker is liable to misdescribe and misapply his own rules. Positively, it should be a speech act that makes sense as part of the whole body of the actions of the speaker. This of course is a very adaptable criterion and makes the considerations that are available as complex as human life. The idea behind this is that a natural language such as English has rules ready to deal with frequent cases (or cases that are sufficiently similar to the frequent cases), but that the writ of the rules runs out in rare or exceptional cases. This is why it is crucial to consider speech acts involving vague concepts that are part and parcel of everyday life, instead of forcing a speaker through an artificial sorites series. I doing so, I will invoke speech act theory as a tool of describing standard use.

This essay features two parts. In the first, I consider non-compound statements such as "a is F", in the second, compound statements, in particular conditionals of the form "If a_k is F, then a_{k-1} is F". I begin by taking a closer look at borderline cases concerning "a is F". In such cases, the rules do not decisively regulate whether a case should be called F or not-F, and it is permissible to go both ways, though not at the same time (section 8.1).

This double permissibility makes room for a couple of ways in which borderline cases can occur in actual speech: As barely noticed *ad hoc* decisions, as a problematisation of the unclarity of a speech act, or as a rejection of a speech act as infelicitous (section 8.2). Then I consider whether speaking of borderline cases introduces a third category that leads to the problem of higher-order vagueness (section 8.3), and in the wake of that discussion, consider the forced-march sorites which is crucially connected to the former problem (section 8.4). The second part is concerned with compound statements. In section (8.5), the speech act theoretical treatment of occurrent borderline cases is applied to compound statements about borderline cases, notably penumbral connections. This leads to my discussion of the conditional sorites (section 8.6), where I claim that the indeterminacy about borderline cases also affects rules in the form of conditionals. Then I compare my solution to the structurally analogous solution of Ben-Yami, and discuss unproblematic replacements of Tolerance (section 8.7). The findings of my discussion then are summarised in section (8.8), where I consider the attractions of the sorites and why we are prone to accept Tolerance as a rule, and where forcing our way through a sorites gets its appeal from.

8.1. Borderline cases

Vague expressions, in particular predicates, have borderline cases even if they are fully contextualised in terms of comparison class, contrasting cases, or setting. For instance, "small heap" has borderline cases even if we give a lot of additional information: It's a heap of spice, curcuma, in a backstreet of Mumbai in the 1990ies, for a small spice vendor. That is, according to the type-meaning of "small heap" so specified, there are heaps for which it is not mandatory to favour "not small" over "small" (or vice versa). Rather, it is permissible to call them "not small", *and* permissible to call them "small". In other words, in my view, appealing to context does nothing to solve the sorites paradox. Nevertheless, context comes into play in another way. The strategy of the not-a-rule approach of declaring Tolerance as not a rule *cannot* be duplicated for statements about borderline cases. Empirico-contingent statements about borderline cases are a real possibility. While it is permissible to go either way in a fully specified normal context, it is not permissible to go both ways *in the same situation*. Accordingly, we may introduce something called "borderline context" as ranging over this same situation. And it is not permissible to call a heap "small" and "not small" in the same borderline context.

The notion of borderline context is inspired by the idea of a conversational score along

the lines of Shapiro, who takes the idea from Lewis (Shapiro 2006, pp. 12f). The core point is that within a conversation, its participants remember things: disambiguations "You are eating a rocket? – I am talking about salad", specifying the bearer of a name "Adam, as in the guy we met at the casino", specifying a context "Young for a post-doc", anaphora "Do you know Peter? He's very kind", standards of exactness "France is a hexagon. – Then Italy is a boot", presuppositions "Has John stopped to beat his wife?". All these things can be written into the score book of a conversation (or a monologue), but they can also be erased from it "Now I am talking about the other Adam." In all these cases there are options that can be specified (or assumed) in a conversation, and kept track of, and later dismissed. In borderline cases, the options are calling a particular case F or not-F. Once a heap is said to be small, this is kept in the score book until actions are taken to modify that verdict or the conversation is over. As in other cases, to contradict oneself in a borderline case is unacceptable "But you just said the heap was small!". As in other cases, one can change one's mind, or use contradictions to make a point.

Two people verbally disagreeing whether a heap is small or not does not always amount to a genuine disagreement. A prime example of a merely verbal disagreement is the case of context-sensitivity. "This heap is small and not small; small for a backstreet vendor, but not small for a wholesale trader." Disagreements about borderline cases have a certain similarity to contextualised mere verbal disagreements. It is permissible to call the same heap "not small for a backstreet vendor" and "small for a wholesale trader". Equally, it is permissible to call the same heap "small" and "not small" *in different borderline contexts*. Because borderline cases are up for grasp, it is pointless to argue about them for internal reasons. Of course non-vague predicates such as "taller than 90 % of the group" are also context-sensitive, and have a normal context (which group?). But they do not give rise to borderline contexts.

In the case of the law, however, the situation is quite different. As far as I know, all systems of law involve precedences. These are cases where a judgement has been administered, and which in turn serve as justification for future cases. In that way, if a precedence judgement concerns a borderline case, this judgement is not dismissed later, but rather kept track of. Therefore, in law, there are no borderline contexts. For the question which judgement should prevail, all sorts of legal considerations will come into play. Was it manslaughter or murder? I regard this as a special case without delving more into details.

According to the not-a-rule approach, the extant rules for the thus contextualised expression "small heap" do not regulate the location of the boundary between heaps that are small and heaps that are not small. Moreover, the rules that are in force are most prominently based on paradigms. And these rules pull in different directions. The borderline cases are relevantly similar to paradigms of small heaps, but also relevantly similar to paradigms of heaps that are not small. In other words, the reasons for applying "small heap" pull with roughly equal force in both directions. This is why the rules do not force our hand to go either way, and why going both ways is acceptable. In that way, the lack of a decisive rule allows the speaker to decide for himself in borderline cases, thereby establishing what I have called a "borderline context". However, that one can go either way in borderline cases is not a rule, but rather a consequence of the rules that are in force. Indeed, speakers are typically unaware that they can go either way and in some cases quarrel about the right verdict. The rules do not legislate a clear verdict in those cases, and this is what makes it permissible to go either way. Speakers may be criticised for their borderline applications, but the grounds of the criticism are not stronger than those of the application itself.

A borderline case, the way I use this expression, is a case where it is permissible to judge F and it is permissible to judge not-F, and the double permissibility is not due to a difference in normal context. Moreover, in borderline cases of F, it is indeterminate whether F applies or not *because* there are reasons that pull in both directions. In that sense, Fine's example of incomplete definition (numbers larger than 17 are nice, numbers smaller than 13 are not nice) does not give rise to borderline cases: There are no conflicting reasons or rules; there are simply no rules that would say anything about the numbers from 13 to 17 (Fine 1975). Nevertheless, in such a case it is indeterminate whether 14 is a nice number. Moreover, two kinds of borderline cases can be distinguished. Borderline cases of one-dimensional vagueness, and borderline cases of multi-dimensional vagueness. In the one-dimensional case, only one dimension is involved: the numbers of grains for "heap", similarity of shade for "red", numbers of hairs for "bald". The objects to which one-dimension vague concepts are applicable can be arranged in respect of similarity or quantity in a dimension δ. For a one-dimension vague concept F, on one side of the dimension there are clear cases of F, on the other side clear cases of non-F, and in the middle there is a transition zone where it is unclear whether F applies or not. In contrast, multi-dimensional vagueness consists of a variety of competing dimensions where borderline cases are the result of the conflicting pull that *different* dimensions

exert. "He is kind of nice, but also kind of not nice. He went out of his way to help me get an apartment, but then failed to greet me on several occasions, etc." (cf. Alston 1964, p. 87). Unless otherwise noted, I use "vagueness" in the one-dimensional sense.

The extension of a (one-dimensionally) vague predicate F in a specific normal context can then be described as follows. Ordering the objects it applies to according to its underlying dimension, the extension clearly includes cases of F on one side, the anti-extension clearly includes cases of non-F on the other side, and in the middle, there are cases for which it is unclear or indeterminate whether they belong into the extension or the anti-extension; these have borderline status.

At this point, there is some pressure to say that the borderline cases have a truth-value different from being true and being false. After all, we are neither able to clearly assign them the value "true", nor the value "false". Therefore, one could think that statements about borderline cases are neither true nor false. I discuss three ways to go on from here. The first is to treat "neither true nor false" as a third-value. This amounts to there being a third row called "indefinite" which appears in truth-tables, there being an operator "It is indefinite that ..." which can be applied freely to any sentence, and statements about borderline cases being used without restriction in inferences. I don't think that this is a promising route for two reasons. First, there is little evidence in the actual use that the speakers operate with a full-fledged third truth-value. For instance, there is no obvious name for it: "indefinite", "indeterminate", "unassigned", and "neither true nor false" are all equally inappropriate as a name for a full-fledged truth-value. Second, similar reasons to introduce a third truth-value could speak for further truth-values, since there seem to be borderline cases between true and indefinite. And this road clearly leads to over-precision in the form of degree theory.

The second option is to treat statements about borderline cases as having a truth-value gap. Accordingly, a statement that is neither "true" nor "false" is a statement *without* a truth-value. This view has some affinities to what Tye and Hanfling propose. Officially, Tye (1994, p. 194) endorses truth-value gaps: "The third value here is, strictly speaking, not a truth-value at all but a truth-value gap." However, we receive no explanation of the difference between truth-value gaps and truth-values. Moreover, Tye immediately continues to provide truth-tables including the value or gap "indefinite" and embraces the consequence that the Law of Excluded Middle is no tautology according to his truth-tables. Therefore, contrary to his declaration, Tye treats truth-value gaps indeed as a third truth-value (at least according to the distinction that *I* draw). The closest

proposal is the one of Hanfling (2001, p. 34–35). While he does not use the expression "truth-value gap", in his proposal a speaker should say in borderline cases that it is "neither 'true' nor 'false' ". As he explains, this is not to be equated with "neither true nor false" and does not not amount to a third truth-value. "The latter represents a claim, the former merely an abstention". He also indicates that this abstention can be elucidated by comparison to Strawson's category of presupposition (without doing so). I will try to fill this gap by bringing in speech act theory. The core idea is that gaps can be treated as infelicitous statements; they are attempts at making a statement, but fail, and are for this reason void. Infelicitous statements about borderline cases are withdrawn from circulation. The advantage of this is that they have no influence on the logic for vague concepts.[1]

However, there is an important caveat to make: A statement about a borderline case does not *automatically* lack a truth-value. After all, if I am right, it is permissible to go either way. And this is based on the observation that for Berta, there may be an overlap of verdicts in a given normal context which gives us no reason to criticise her. This gives rise to the third option, and complicates both what the speech act theory has to say about borderline cases, and the relation between meaning and extension (reference, truth). Roughly speaking, if all other speakers in a given situation go along with the decision of Berta about a borderline case, this statement has the truth-value that Berta has assigned to it. However, if her decision is contested, there is no accepted *ad hoc ruling*, and her statement lacks a determinate truth-value, rendering it infelicitous. (I will discuss more cases below.) The point here is that the type-meaning of F leaves it open to go both ways in borderline cases. To make an *ad hoc ruling* for them does not clash with the established type-meaning. This fits well with Strawson's insistence that logic applies to statements (uses of sentences), not type sentences. One reason for this are indexicals such as "I" and "you", which have a reference only in the context of a statement. And another is that *ad hoc rulings* for borderline cases are not dictated by type-meaning. In other words, for statements, a borderline case either receives an *ad hoc ruling*, or the statement itself is void. In that way, vagueness is compatible with classical logic for statements that are not infelicitous due to a truth-value gap.

[1] Manor (2006, p. 173) and Keil (2010, p. 81) contain rudiments of such pragmatic considerations. For Manor, a speaker would mislead a hearer if he would give the order "Serve the bald men tea" in a situation with occurrent borderline cases of "bald". For Keil, the drawing of sharp boundaries is a task of speakers, but not of predicates.

8.2. Applying speech act theory

What is the status of statements (and other speech acts) about occurrent borderline cases? The third proposal that I discussed above suggested that different cases need to be distinguished. In some cases, the speech act is felicitous and does not differ from speech acts that do not touch upon occurrent borderline cases. In other cases, the speech act is infelicitous and is at least temporarily rejected pending further clarification. In other cases still the speech act addresses and problematises the unclarity of borderline cases and thereby ascends to a meta-level: "What do you think? Is this red?" In what follows I will situate this analysis of statements about borderline cases in relation to speech act theory.

The founder of speech act theory, Austin (1975, Lectures II, XI), distinguishes between two broad categories of infelicities: abuses and misfires. Speech acts that are abuses are flawed, those that misfire are void. For instance, a promise given without the intention to live up to it is dishonest and therefore flawed, but the promise is not void. In contrast, the promise to find a murderer is void if no murder has taken place. The speaker has made a promise, but the promise is without effect. The infelicities I am concerned with are of the latter category, they are misfires. One instance of a misfired speech act is the case of a captain stranded on an island who marries a couple. Since he is not on his ship, he does not have the authority to go trough with the marriage ceremony. His speech act "I hereby declare you husband and wife" is without effect (they are not married), or void.

To say that a speech act is void (a misfire-infelicity) does not mean that it is without any effect. Clearly, the captain has emitted a sound, used common English words, and put them together in a way that forms a grammatical English sentence. He also manages to refer to the soon to be married by the word "you". His speech act can be reported with the expression "He said that ...". Moreover, for speech acts such as questions, orders or statements, they can be reported with the appropriate verb "He asked me ..." even if they are void. And in the case of the captain, we may say that he attempted or purported to marry the couple.

An infelicitous speech act gives the hearer licence to criticise the speech act (and the speaker). The criticisms are as diverse as the infelicities. A promise may be criticised as insincere, an order may be rejected on the grounds that the speaker has no authority, a speaker may be reprimanded for making a statement when he is not in the position to know. The infelicities that are closest to borderline cases infelicities are unclarities

(which would fall under the Gricean maxim of manner). Unclarities call for clarification *before* a statement can be evaluated for its truth, or a question can be answered. The testimony "Jack the ripper ... he ... (mumbles, cries)" cannot be evaluated for its truth because it is incomplete and partially (acoustically) incomprehensible. The question "Do you think Peter is young?" cannot be answered without specification of context: "Well it depends. He's young for a politician, but not a young member of society any more." Or the statement "The red haired boy is the worst rascal of your class." may be criticised in the following way: "Whom do you mean? I don't have a red haired pupil." Similarly, the tale "Yesterday, Rob has found a blubb on the street." is pending clarification on what a blubb is before its truth can be assessed. These criticisms do not lead to a breakdown of the machine. Rather, they are interjections that try to keep the boat afloat.

The last case brings me to Strawson's category of presuppositions (1950, 1952, p. 175f), which can be compared to borderline infelicities in a fruitful way. Both are sources of truth-value gaps. If a suspect is asked "Where did you hide your weapon?", and he is not the murderer and didn't have a weapon, he cannot truthfully answer the question. He needs to challenge the presupposition of this mixed speech act: "I never had a weapon! I didn't harm anyone!" Similarly, if the policeman writes in his report:

5) The suspect has thrown his weapon into the Rhine.

he has not merely written something false, but something that requires a more basic admonishment: It is neither true nor false and should drop out of consideration. Since the suspect didn't have a weapon, the question whether he has thrown it into the Rhine does not arise, the speech act (5) is void.

The failure of a presupposition has different consequences for different speech acts. In the case of a statement, the speaker has failed to state anything that has a truth-value. An order such as "Bring this book to the present king of France!" is infelicitous in that it cannot be carried out. Depending on the power relations, the respondent could reject the order as infelicitous by exclaiming "There is no present king of France!". However, that does not mean that no order has been given. We may find a shaking palace barber who has been ordered by the king to shave all and only those palace dwellers who do not shave themselves. A question that is infelicitous because of presupposition failure cannot be answered in the usual way without accepting the presupposition (this holds for yes-or-no questions, where, when, why, how, in what way). In order to avoid accepting

the presupposition the interlocutor needs to remain silent (though even that may be construed as a silent acceptance of the presupposition) or challenge the presupposition.[2]

Before we come to the infelicity of statements about borderline cases, let me mention another important feature of them. Borderline cases are easy to construct, but are an infrequent feature of our ordinary linguistic practice. Vagueness is ubiquitous, but occurrent borderline cases are rare. There are two reasons for this. For one thing, many borderline cases are potential rather than actual: There are no berries that are half raspberry, half strawberry, and all the development stages between Australopithecus and Homo Sapiens have died out. For another, occurrent borderline cases, with their threat of unclarity and infelicity, are impractical. This gives the speaker a motivation to avoid them. But it also offers an incentive for a language to grow to accommodate occurrent borderline cases that obstruct the pursuit of practical endeavours. An apprentice bringing back a bucket of paint that his boss calls "blue", but that he thought is green is wasted money and effort.

A mature language offers many devices to avoid occurrent borderline cases. The painters could use a sample card of a colour scheme with more divisions. Modifiers such as "small", "large", etc. can be used to avoid borderline cases. Another device is to specify an object of comparison (or comparison class). If a comedy director requires a couple of actors that are kind of bald but not completely bald he might tell his agent "I need actors that are bald just like Gorbachev". Of course, these are ways to make what is said more specific, and to avoid falling between two chairs by pushing in a third chair, but not of getting rid of borderline cases completely. There are still borderline cases for "bald just like Gorbachev". Moreover, the modern language affords many conceptual systems that can take over if other rough and ready conceptual distinctions are not precise *enough* for the purpose at hand. Instead of talking of "young" and "old" books, we might specify their age in years. Of course, such devices allow not only to avoid borderline cases, but also context-variance, unspecificity and ambiguity if they give rise to unclarity (cf. Keil 2010a, pp. 96f).

The fact that occurrent borderline cases are infrequent does not resolve their status. One can distinguish three broad ways in which occurrent borderline cases in speech acts may be treated. First, they are regulated by an *ad hoc* decision. Second, their status as borderline cases is problematised and brought to attention. A common way to

[2]For an excellent description of the behaviour of presuppositions in compound sentences, see Karttunen 1973.

do this is to assert negated instances of the Law of Excluded Middle or of the Law of Non-contradiction. Third, the speech act is rejected as unclear and infelicitous. Such a rejection, however, need not be the final word, but rather an invitation to clarify, as in the following exchange: "Send this letter to Jeffrey Lebowsky." – "I don't have his address." The hearer's rejection is also a plea for more information. A similar issue comes up for the order

6) Scoop up all small heaps in this room.

Whether the order is felicitous depends on the situation of the room. Imagine that the room is full of heaps that differ only slightly in their height. In that case, unless the speaker has specified what he means by "small heap", the order is infelicitous. In contrast, if there are two groups of heaps that clearly differ in their size, then the order is felicitous and can be carried out without further ado.

Regarding questions, there is a difference analogous to the difference between an empirical statement and a rule. Questions may ask for factual information, or for the rules of an expression, for instance the applicability of a predicate.

7) Is this heap small?
8) Is this guy a rascal?

A speaker who is competent with the use of the expression "small heap" has the ability to recognise small heaps on sight, barring bad lighting conditions, etc. In contrast, whether a child is a rascal cannot be seen on first sight (even though there may be some characteristic leads). Accordingly, if a speaker asks (7), probably, it is a question for the applicability of the predicate, and if he asks (8), it is probably a factual question. Consider again the situation of the heaps in room.

9) How many small heaps are in the room?
10) Are there thirteen small heaps in the room?

Clearly, both questions may receive a factual interpretation, for instance if the speaker is not in the room, or if there are two groups of heaps which differ significantly in size. However, if there is a steady progression in the sizes of the heaps, and the speaker is fully aware of that, the questions may become disingenuous and infelicitous. Knowing what "small" and what "heap" mean do not equip a hearer with the rules to answer

those questions, he is asking the hearer something he knows the hearer cannot answer. In that case, it is an infelicitous question (perhaps as part of a power play). But perhaps the question can be regarded as an invitation to make an *ad hoc ruling*, and that would be felicitous. Or (10) can be regarded as a guess concerning the *ad hoc ruling* of the hearer (perhaps as a reaction to (9)). A sign that these questions are not meant as an invitation for an *ad hoc ruling*, or a guess concerning the other's *ad hoc ruling* is if the speaker *insists* on an answer. But in that way he betrays misunderstanding. There is only an answer if we stipulate one, not by virtue of the standard type-meaning of "small heap". A hearer then is justified in rejecting the question: "Why do you want to know?"

Often, utterances that look like statements may also express *ad hoc rulings*. For instance, in the case of the steady progression,

11) There are 13 small heaps in this room.

may be regarded as stipulation of what the speaker regards as small heaps, or what he is inclined in this situation to regard as a small heap (with no claim of reapplicability).

In that way, simply saying about a borderline case of F "This is F" does not lead to a truth-value gap. The default interpretations avoid such gaps. Nevertheless, we might reply to "Is this F?" with "Kind of" or "Kind of yes, kind of not". In such cases, neither the question nor the reply is infelicitous, but hearer and speaker come to an agreement that one could go either way in that case. They do not ascribe any determinate truth-value to "This is F". Do we therefore have here a case of a felicitous statement which lacks a truth-value? But neither of the speakers asserts "This is F". They agreed that it's kind of F, and kind of not-F. To come back and state "This is F" would thereby go against their agreement and be for this reason infelicitous. It is a way to acknowledge the indeterminacy.

Even though (11) has potential borderline cases no matter whether there are two distinct groups or a progression of heaps in a room, they are occurrent only in the second case. And only occurrent borderline cases threaten to make a statement unclear. Moreover, there are many interpretations of a communication situation, and many devices, which allow to deal in a constructive way with occurrent borderline cases. As a result of this, it is rare that occurrent borderline cases become problematic. Nevertheless, in such cases the speech act is infelicitous, and as such has no ramifications for compound sentences. An infelicitous statement does not become felicitous by combining it with felicitous statements.

8.3. Higher-order vagueness

In one respect, what I have said so far does not differ much from other solutions to the sorites: There are clear cases of F, clear cases of not-F, and a borderline region in between. I claim that borderline cases are cases where truth-value gaps *may* occur, but need not (due to speaker's *ad hoc rulings*). This is a feature of the type-meaning of vague predicates: It is permissible to go either way in borderline cases. Have I thereby unwittingly replaced one sharp boundary between heaps and non-heaps with two, between heaps and borderline cases, and borderline cases and non-heaps?

This is the problem of higher-order vagueness that besets those solutions that introduce a third category on top of truth and falsity, or of being F and being not-F. The idea is that a sharp boundary between truth and falsity can be avoided by separating the two by cases that are, say, indeterminate. But the problem seems to recur, because now there are two sharp boundaries in a sorites series, between "true" and "indeterminate", and between "indeterminate" and "false". In that way, the same reasoning that supported the introduction of a third category supports the introduction of an infinity of categories. This has the disadvantage of not getting rid of sharp boundaries, but multiplying them, and of introducing over-precision through the infinity of categories.

However, this version of higher-order vagueness can be sidestepped by not introducing a full-fledged third category, but only, so to speak, a half category. This is what the not-a-rule approach proposes. The extensions and anti-extensions of vague terms have to be regarded as vague sets. There are cases that clearly belong to the extension of F, and then there are cases where it is indeterminate whether they belong in the extension or anti-extension. Moreover, "indeterminate" is not a full-fledged truth-value. On the contrary, statements about borderline cases may be rejected as infelicitous, and mark an unclarity in speech. In that way, no full fledged category of "indeterminate" separates truth and falsity, but the applicability of "true" and "false" becomes indeterminate in the borderline region. In effect, this treats indeterminacy as irreducibly indeterminate. It cannot be reduced to determinate ascriptions of indeterminacy to individual statements.

Nevertheless, someone might ask where "true" gives way to "indeterminate" in a sorites series. And the answer is: "Somewhere in the middle". This answer may not satisfy the interlocutor and he may resort to go case-by-case through a sorites series. By doing that, he connects the problem of higher-order vagueness with the forced-march sorites.

8.4. The forced-march sorites

In the forced-march sorites, we are confronted with a sorites series and are asked to give a verdict for each individual case. For example, after each removal of a grain from an agglomeration of a lot of grains we have to give a verdict whether it is a heap or not. By so proceeding, the number of grains dwindles and eventually no grains are left. If we are cooperative and stick to the terms of the cooperation, the same dilemma as in section (7.1) is forced on us without that our interlocutor is committed to any premises: Either we change our verdict at some point and are thereby committed to sharp boundaries, or we try to avoid sharp boundaries, but have then to say that there is a heap even if only a few grains are left. There are many versions of that problem. Instead of giving a verdict whether the vague predicate F applies or not, the task could be to assign a truth-value to statements of the form "This is F", or to statements of the form "It is true that this is F", etc. One noteworthy variant is Horgan's (1994, p. 173) forced-march sorites where the interlocutor is asked whether the two statements of the form "This (i) is F" and "This (i-1) is F" have the *same* truth-value or semantic status (regardless of how many truth-values, etc. the interlocutor wants to have at her disposal). Note also that the form of the version of the ancient sorites we know is a series of questions that the interlocutor has to answer.

> Wherefore I say: tell me, do you think that a single grain of wheat is a heap? Thereupon you say No. Then I say: What do you say about 2 grains? For it is my purpose to ask you questions in succession, and if you do not admit that 2 grains are a heap then I shall ask you about three grains. Then I shall proceed to interrogate you further with respect to 4 grains, then 5 and 6 and 7 and 8. (Galen, cited after Barnes 1982, p. 33)

The forced-march sorites can also be regarded as a sort of answer to my claim that borderline cases very rarely come up in ordinary discourse. Even if that is true, sorites series can be constructed, and borderline cases inevitably will come up in such series. Then the question regarding the correct verdict in borderline cases reappears.

The main characteristics of a forced-march sorites as I conceive it are that it is both comprehensive and detailed, and that it requires a certain form of reply. A description can be comprehensive without being detailed. A comprehensive description conveys the big picture and does not leave out any case that is relevant to the inquiry at hand. It

is comprehensive to say that collections (of the right form) of a few grains are not a heap, collections with a lot of grains are a heap, and collections with a medium amount of grains are borderline cases. But in order to be detailed, one would need to say for each relevant permutation of the case whether it is a heap or not. One way to achieve a comprehensive and detailed perspective is to consider each case of a sorites series one by another. The particular order of that exercise does not matter, nor does the particular form of the statement (or speech act) that is considered. In that way, we can both have forced-march sorites with "a is F" and with "If a_i is F, a_{ii} is F".

But the crucial issue of the forced-march sorites is whether the range of answers is restricted. If they are restricted, this invites doubts of what to make of the answers that the respondent gives. If we hold a gun at a speaker's head and force him to say "I am too stupid to know the answer!", it doesn't follow that he *is* too stupid to know the answer. Or if the answer-options are "green" and "yellow", and there are red patches among the objects of the series, it does not follow that red is the new yellow. Or to refer to an example of experimental philosophy, where the respondent is asked to classify a decision of a manager as either "intentionally harming the environment" or "unintentionally harming the environment", but without the option that the manager is "prepared to accept to harm the environment" (Knobe 2006, p. 205). Equally, if the available answers in a sorites series are only "true" or "false", the respondent is by that fact forced to draw a sharp boundary. But then, clearly, what he actually does answer does not reflect what he *would* answer if he had more options. Now, in Horgan's version, the response options are numerous and only restricted in two ways. i) The respondent should give *some* answer in *each* case, and ii) his answer should be a "semantic value".

Concerning borderline cases, the not-a-rule approach is committed to type-indeterminacies and to truth-value gaps and infelicities in speech. Accordingly, the same problem is a threat. *Prima facie*, if I am forced into such a sorites questioning, I would not fare better than a proponent of any approach that introduces a third category. No matter whether I choose to say "true" and then "false", or "true" and then "gap", my answers will show the pattern of a sharp boundary if I decide to evade the paradox. Perhaps the best I can do is to stick to "true" and be committed to paradox in protest. Now, if we drop the last two requirements (apart from the requirement not to walk away), I would say:

> My claim is exactly that there is no sharp boundary between "true" and "false", or between "heap" and "not a heap". That is why I cannot give you an

> answer for *every* case, though I can answer a few times "true" at the beginning of the series, and a few times "false" at the end of the series. But there is no sharp boundary to which I can, upon reflection, agree. In ordinary discourse, the occurrence of a borderline case often counts as a communicative error. There are many devices to avoid them, and if a speaker's speech act is marred by the occurrence of borderline cases, his interlocutor is licensed to criticise these speech acts as infelicitous. In the case of a statement that is infelicitous due to borderline case occurrence, the statement lacks a truth-value, and the ball is back at the speaker because the hearer cannot do anything with his statement (in the case of longer speeches, he can pick out the parts that are not marred by borderline case occurrence, though). The reason for this is that the rules that are in force for the concept of heap say something about its form, its possible materials, its relations to other concepts such as stack, what are clear cases of "heap", and what are clear cases of things that are not heaps. But there is no rule that regulates the boundary. And that boundary has no practical purpose, anyway.

In that way, the correct reaction to the forced-march sorites is to insist on the absence of a sharp boundary. There are Fs, and there are not-Fs, but there is no first not-F in a sorites series, nor a first case of a truth-value gap. From that perspective, the forced-march sorites is a method of investigation that leads to biased results.

Note that this strategy is not the same as discounting speech acts made in the context of a sorites paradox by a speaker, when considering which rules that speaker is following. In that case, the argument is that one cannot expect from a random speaker that she is able to solve the sorites paradox that has puzzled philosophers for centuries, and that she is liable to make an epistemic mistake. In that case, assertions and rule-expressions made under the influence of epistemic mistakes do not reflect the rules that the speaker actually follows. In contrast, the question here is what a fully prepared speaker (let's call her a philosopher) should and could answer in order to avoid paradox.

What lies behind the idea that certain methods of inquiry lead to biased results is that our language provides the rules for its concepts in broad strokes, leaving unpainted areas on the canvas. Many things remain unregulated, especially details such as the acceptance of one particular conditional, or of a particular judgement of the form "F(a)". What is notably unregulated is the boundary between F and not-F. Speakers are perfectly

aware that a rule such as "The smallest heap has 42 grains" does not play a role for the meaning of the term "heap". Now, the point of confronting a speaker with a sorites series is to make him give a verdict in each case. The thought is that in that way he is forced to commit himself to sharp boundaries. And in a way that is true. If he agrees to play that game and wants to uphold the distinction between Fs and not-Fs, he has to single out one grain that makes the distinction. But peddling through a sorites series is not one of the situations for which the rules that are in force prepare a speaker. The rules give him no guidance in such cases, except that detailing out the canvas changes the original picture. To identify a sharp boundary changes the concept in question. For ordinary purposes, it is enough to know paradigm cases of heap, that the features of form and size are relevant, and that one can take away a few shovels of grains from a heap without turning it into a non-heap, but that one should not take away too much.

In other words, making a distinction between heap and non-heap based on a very small difference would introduce a novel, but related concept. This is often called the "absurdity of a sharp boundary". However, one needs to be aware of what exactly is absurd. It is absurd to take the altered concept to be the original one, not to draw sharp boundaries and introduce a novel concept (cf. *PI*, §§ 68–69). Such a move is perfectly legitimate if it has a purpose, such as the legal sharpening of "adult", the organisation of colour pigments in a shop, or even an *ad hoc* explanation of an order "Scoop up the small heaps!". The only thing that needs to be kept in mind in such cases is that we are now dealing with two related, but different concepts.

The strategy of rejecting the forced-march sorites as a biased instrument relies on the correctness of the analysis that for vague predicates, the boundary is unregulated, and therefore not sharply bounded. If there are other methods to show that vague concepts are sharply bounded, the issue of the bias of the forced-march sorites could be sidestepped. However, it seems to me that the forced-march sorites is the crucial instrument in the discussion. Have we therefore reached an impasse? A proponent of the not-a-rule approach may claim that the forced-march distorts the concepts in question, while his opponent may admonish him for refusing to answer crucial questions. And both have a point. If, for vague concepts, the writ of the rules runs out in rare and exceptional cases, a forced-march is bound to misrepresent that fact. If, in contrast, all these cases are regulated, a forced-march does not misrepresent this feature of vague concepts. In other words, both views on the present or absent bias of the forced-march presuppose that their view on the regulatedness of vague concepts is right.

There are two considerations that I would like to bring in at this point. First, the other existing proposals for solving the sorites each are unsatisfactory in their own right. This of course requires a lot of work that I will not attempt here. But in the same vein, there is a systematic problem that besets solutions that do not regard indeterminacy as an integral and irreducible part of, well, indeterminacy. This is the problem of higher-order vagueness that I spelled out above. Any attempt to capture the phenomenon by introducing a third category that is not itself vaguely bounded is on a path to further sharp boundaries, and eventually, over-precision. That is why an attempt that keeps vagueness vague should be considered seriously, and does have a systematic advantage. Second, why should we accept the demands of the forced-march sorites? It demands to give a determinate description of indeterminacy by ascribing a determinate semantic status to each statement. But what could serve as a justification of this demand? At worst, it is merely a logical prejudice. Indeed, it has its roots in Frege. And when Frege demands that concepts be sharp, he envisions an artificial language as a tool for thinking. But when considering English, we consider a natural language that has its natural and cultural history, and that has grown into the various roles and purposes for which it is used. It should come as no surprise that such a language has no answers to arcane questions.

8.5. Compound statements and penumbral connections

So far I have discussed non-compounds statements such as “This is a heap.” What shall we say about a compound statement where one of its parts is felicitous, and another is infelicitous? One could make a case that a three-valued approach (or another alternative logic) has an advantage here, since it allows to compute the truth-value of compound statements, not least among them the laws of logic. However, I argue that to ignore the lessons of speech-act theory leaves us with too coarse a cleaver. I do that by advancing analyses of a number of compound sentences and corresponding statements. The gist of my speech-act theoretical approach (with truth-value gaps) is that infelicities drop out of the picture. When starting with a statement that has already been rejected as infelicitous, nothing is gained by joining it with a felicitous statement, since that does not remove the infelicity of the first statement. Indeed, when confronted with speech where parts of it are felicitous, other parts infelicitous, a good start is to ignore the infelicitous parts, and to try and clear them up later.

The same speech-act categories that are applicable to non-compound statements are also applicable to compound statements. They may be a) outright infelicitous, b) a problematisation of the lack of rules in borderline cases, or c) an *ad hoc ruling* that papers over the indeterminacy of the type meaning. In the case of logical laws, a fourth category comes into view: d) a reminder of the meaning of a term, or of what is part and parcel of making a certain speech act (such as the Law of Non-contradiction for statements). Other categories are applicable as well, but these are the main ones.

Concerning the four main categories applicable to statements about borderline cases, the consequences for truth are the following. a) If the whole statement is infelicitous, it has no truth-value. b) If it is a problematisation, it does not have a truth-value, but is a different type of speech-act altogether. In such cases, the indeterminacy of the type-meaning comes to the fore. One standard way of problematisation is to negate a law of logic. But it is the very status of these laws as fundamental for making statements why their negation can be used to flag an unclarity. c) If it involves an *ad-hoc ruling*, it has a truth-value, and its truth depends on the state of the world, the existing body of rules, and the *ad-hoc ruling*. d) If it is a reminder of the meaning of a term, it has a truth-value in the same way as uses of analytic sentences have a truth-value (cf. Schroeder 2009).

The simplest case is when two non-compound statements are joined together by a logical connective.

12) Either Tek is tall or Robert is a rascal.

However, the apparent conjunction of a felicitous statement with an infelicitous one does not result in a compound statement whose felicity is to be determined. Rather, it results in speech where one part cannot be assessed for truth, and the other part is a non-compound statement. Unless the conjunction establishes an *ad hoc ruling* for the borderline case. For instance, if we know that Robert is not a rascal, then an utterance of (12) suggests the *ad hoc ruling* that Tek *is* tall.

However, there are statements where the infelicitous part is not so easily discarded. In the case of quantifications, discarding indeterminate cases would teach us to not to make quantifications at all, which would be a real loss. So how we should treat occurrent borderline cases in statements such as the following?

13) All swans on the lake of Zurich with a completely white plumage are adult.

For one thing, it is clear that this quantified statement could be false. While the existent swans typically feature a white plumage once they are adult, it is conceivable that there may be swans which we would call "adult" due to criteria such as size, bodily development, and age, but which have a half-grey plumage. Maybe there are no occurrent borderline cases. But what if we encounter a swan at the lake of Zurich that is a borderline case for both "white plumage" and "adult swan" (a few tiny grey spots, not completely grown)? The charitable thing would be to count this swan as non-adult, that is insert our own *ad-hoc ruling*, and therefore to regard (13) as true. (Or to discount the tiny grey spots and call its plumage "completely white".) Of course, there can also be conflicts of charity, as when somebody would say:

14) Some adult swans on the lake of Zurich have grey spots in their plumage.

Perhaps the disputants can agree that these things are too unimportant to argue about, or perhaps they precisify and operationalise what they mean by "completely white plumage" and "adult". Or they disagree, but then their disagreement is one where the decision regarding borderline cases diverges, and is therefore a mere verbal disagreement.

So far I have discussed compound statements whose parts have a contingent relation. Whether Tek is tall does not depend on Robert being a rascal. However, if "This patch is orange" is true, then "This patch is not-orange" should be false. Fine (1975, p. 270) coined the term "penumbral connection" for compound statements that have a logical (or conceptual) dependency that should hold even in borderline cases. The foremost example for penumbral connections are (instances of) logical laws, but also conceptual connections such as if Tek is tall, and Tok is taller than Tek, then Tok is tall, too.

Supervalutationists such as Fine see the treatment of logical laws as a topic where supervaluationism has an advantage over three-valued logic. The Law of Excluded Middle comes out as true in supervaluationism, whereas for three-valued logic, it is indeterminate. How does my truth-value gap plus speech-act theory account deal with them? For one thing, the idea that occurrent indeterminacies are infelicitous enforces the truth of the laws of logic, since infelicitous statements drop out of the picture. First clarify your statement, then we can discuss its truth. For another, the distinction between type-sentences and uses of sentences complicates the picture. Type-sentences have no truth-value, and the laws of logic (as type-sentences) may be used differently than to make a declarative speech act, viz. a statement.

I begin with a relative of the Law of Excluded Middle (and its negation).

15) The cap he is wearing is either green or blue.
16) The cap he is wearing is *neither* green *nor* blue.

Assume that the speaker observes somebody from afar, which makes it difficult to discern the colour. By uttering (15), the speaker narrows down the possibilities, because he excludes colours such as orange, yellow or brown. This use is in none of the above categories, and it has a truth-value. In a different situation, the same sentence can be used as a reminder (d) of a feature of the meaning of "green" and "blue", namely that they exclude each other (by emphasising the "either"). Does this rule also cover borderline cases? Yes, because in case of verbal disagreements, there is pressure to find harmony. It's not acceptable that one speaker says that it's blue and another that it's green. And it's also better to agree on a stipulation than to stipulate differently.

One use of (16) is to make an informative albeit unspecific statement concerning the colours of the cap (none of the categories, truth-evaluable). For instance, (16) is true if the cap is orange. Another use is a problematisation (c) in the case of borderline cases between green or blue. Assume that it's clear to everyone involved that the cap has no colour other than either green or blue. But in such a case, since its use denies the obvious, it consciously breaks the rule of exclusivity in order to draw attention to the fact that one can go either way. In that way, (16) is similar to "It's kind of blue, kind of green". Similar considerations apply for instances of the Law of Excluded Middle, and their negations.

17) This heap is either small or it is not small.
18) This heap is *neither* small *nor* not small.

A straightforward use of (17) is to remind (d) the interlocutor of a basic feature of a making a statement, and of enforcing the goal of clarity. Equally, a straightforward use of (18) is to bring a borderline case to attention and problematise the case (b).

19) This boy is either a rascal or not.
20) Either the king is responsible for the burning of the city, or he is not.

(19) might be uttered in the face of insufficient evidence as an encouragement to investigate further by reminding the interlocutor of a basic feature of declarative speech (d). Equally, (20) may be used as a reminder (d). But in another case, the use of (20) as a reminder

may itself be infelicitous. For (20), imagine the following case. The king is far away from the city in question and has just received a message that it has been taken. A certain circumstance has enraged him, and he gives a written order to burn it down. A few minutes later, his temper cools and he regrets the order. He immediately sends his best warrior to intercept the messenger, and sends another messenger with an overwriting order. Due to certain circumstances, only the first rider makes it to the city, and it is burned down. Is the king responsible for the fate of the city, or not? Well, in a way yes, because he sent a messenger with a written order to burn it, but in a way no, because he took sufficient action to overwrite and stop the order. In such a case, a statement "He is responsible for the burning of the city" can be criticised as infelicitous because it disregards an occurrent unclarity. Similarly, (20) is not false, and not true, but infelicitous, if it is used to insist that the two options offer a clear choice. However, in other cases it might be used as (19) as an encouragement for further investigation. Compare this with a Gricean case of a violation of the maxim of quantity, where the statement is true but misleading. Bart Simpson talks about his and Lisa's future with a fortune teller. She is the president of the United States, he is an unsuccessful guitarist. When Lisa asks him about her future, he tells her: "You get a government job." This is true, and unambiguously so, but it is not the whole story. "Does Lisa get a government job?" – "Yes." In contrast, there is no simple answer to "Is the king responsible for the destruction of the city?" There is no reason to deny that Lisa is a government employee, but there is reason to deny that the king is responsible for the horrible fate of the city.

Instances of the Law of Non-contradiction can be treated similarly.

21) It is not the case that the king is responsible and not responsible for the burning of the city.
22) The king is responsible and not responsible for the burning of the city.

Instances of the negated Law of Non-contradiction such as (22) can be used for problematising a borderline case. They deny something that is obvious and thereby draw attention to an unclarity. Of course, (22) could also be a summary of the story of a witness, and as such, the contradiction throws bad light on the witness. Unless the witness is able to contextualise the apparent contradiction. Perhaps the sentence (21) could have a use in reaction to such a story. "You have to decide. It's impossible that he is responsible and is not responsible." It seems to be a fundamental feature of the practices of ascribing beliefs to others, and of making judgements about the world that the beliefs and the

judgements should not be contradictory. If a contradiction is uncovered, the suspicion is that it can be accommodated by appealing to some sort contextual difference. If a person is unfazed by an apparent contradiction in his beliefs, this is a reason to doubt his mental sanity.

The laws of logic are not the only cases of penumbral connections. Another important penumbral connection is expressed by Truism Tolerance.

Upward Truism Tolerance: If *n* grains make a heap, *n+1* grains make a heap as well.
Downward Truism Tolerance: If *n* grains don't make a heap, *n–1* grains don't make a heap, either.

These principles sound very similar to the paradoxical Tolerance, but they do not concern the distinction between heaps and non-heaps. As in the case of paradoxical Tolerance, we concentrate on issue of numbers of grains, and take for granted that other dimensions stand still: Grains are added or subtracted in such a way that the form remains the same. Another similarity is that both Truism Tolerance and paradoxical Tolerance can be seen as a universal quantification over conditionals. The relevant conditionals for Upward Truism Tolerance are conditionals such as "If 93 grains are a heap, 94 grains are a heap". Moreover, it is conceivable that individual conditionals might be used to justify and criticise: "Wait a minute. You just said Tom is tall. But then Tek is tall as well, given that he is taller than Tom." However, it is equally conceivable that speakers use the conditionals of paradoxical Tolerance as justification or criticism: "You said that Scott is not tall. But Simon is only a bit taller than Scott. So he's not tall either." So why is it all right to accept Truism Tolerance but not paradoxical Tolerance? I will deal with that difficulty in the next section in full. Here's the difference in a nutshell. In the case of paradoxical Tolerance, any of the conditionals (in the middle) can be accepted individually, but they cannot be all accepted collectively without paradox. In contrast, the conditionals of Truism Tolerance can be accepted collectively, and therefore, the principles can be accepted. In that regard, Truism Tolerance accords with standard use.

8.6. The conditional sorites

In the beginning, I introduced the short sorites, but also made reference to the long version, which uses a finite but large number of conditionals.

23) A collection of 17 grains is not a heap.
24_{i}) If a collection of 17 grains is not a heap, then one of 18 grains is not a heap, either.
24_{ii}) If a collection of 18 grains is not a heap, then one of 19 grains is not a heap, either.
...
24_{k}) If a collection of 9'999 grains is not a heap, then one of 10'000 grains is not a heap, either.
25) A collection of 10'000 grains is not a heap.

The short sorites captures the idea that the addition of one grain *never* makes the difference between a non-heap and a heap in the Tolerance principle. As such it has the advantage of being a concise statement of the paradox. However, if the victim of the paradox is not immediately convinced that he should accept Tolerance, the argument proceeds by considering individual instances of Tolerance. "Imagine a few meagre grains on the floor. If I add one, it still is no heap." This is not the same as saying that the long sorites is the more fundamental. After all, the paradox monger does not appeal to a complete set of conditionals, but only to a few instances. Nevertheless, the long version has the advantage that it makes surveyable how a formalisation of the argument proceeds: As a chain of many *modus ponens* arguments which take the conclusion of the last individual argument as the premise of the next one. And the ten thousand conditionals can be derived via *universal instantiation* from Tolerance.

Dolev (2004, pp. 336–8), who professes to be inspired by Wittgenstein, claims that there is a marked difference between those conditionals with a true antecedent, and those with a false one. Clearly, the conditional (24_{k}) has a false antecedent, and others too. Criticising the formalisation of English "If p then q" statements as material implication, Dolev claims that the only reason to regard conditionals such as (24_{k}) as true is their false antecedent. However, he argues, this reason falls away if they are not formalised as material conditionals. I see two problems with that approach. The first is that the claim that conditionals (in a sorites series) with a true antecedent are true, and conditionals with a false antecedent are not true does not solve the sorites, at least not alone. Now, we are under pressure to isolate the first non-true conditional; and in that way we would be committed to sharp boundaries. The second is that even if we assume that "If p then q" statements should not be formalised as material conditionals, there is still reason to think that they are true. In my favoured alternative analysis of "If p then q" statements, they are partially truth-functional. If the antecedent is true, and

the consequent false, the conditional is false. In that case, we have shown that there is no suitable connection between antecedent and consequent. But in the sorites, no such demonstration is forthcoming, leaving us with no *prima facie* reason for the falsity of the conditionals.

The not-a-rule approach holds regarding Tolerance (and regarding the internal negation of Tolerance) that it is not a rule. What does this mean for the conditionals? It is compatible with a number of options. But it means at least that not all conditionals are rules, and that there is no negation of a conditional that is a rule (which would instate a sharp boundary). I will discuss four options.

i) None of the conditionals is a rule. This would easily stop the sorites, but it has one disadvantage. Speakers are liable to use something very similar to the conditionals above to criticise and justify their applications. "If you call this a heap, you should call that one 'heap', too." Note that in that criticism, no number of grains was mentioned. I don't think that the regular speaker makes up his mind about numbers of grains, nor volume or weight of heaps. Nevertheless, I will roll with the conditionals above because it is easier to handle them. And the use of such criticisms shows what is the basis of counting Tolerance or ten thousand conditionals among the rules of the speaker.

ii) All but one conditionals are a rule. This goes in the other direction and accepts that some of these conditionals play a role in our linguistic practices. It also stops the sorites. Now since only one conditional is no rule, there is only one number of grains for which nothing is determined. The picture looks something like this: The last number of grains for which the rules determine that it is no heap is 41; the last number of grains for which the rules determine that it is a heap is 43; and 42 is the only number of grains which remains indeterminate. Accordingly, there is no real penumbra region, but only a penumbra object. This proposal suffers from too much precision.

iii) The conditionals in the middle ground are not rules. This is a compromise between (i) and (ii). It accepts that the relevant conditionals play a role in the normative practices of the speakers, but also makes room for a penumbra region. But why are they not rules? First, if they were rules, they would distort the vague concept beyond recognition by way of paradox. But a regular speaker of English does not treat "heap" as being equally applicable and not applicable to all cases (the ultimate consequence of the paradox). That is why a fully informed speaker should not accept all the conditionals. A good way to do that is to regard them as not being rules, which does not imply that their negation is a rule (thereby not incurring any sharp boundaries). Second, that the penumbra

conditionals are not rules flows from the idea that the boundary is not regulated. The rough and ready rules for vague concepts never bring the boundary into focus. Together with the equal pull from the paradigm cases on each end, we end up with a penumbra region where we could go either way concerning "a is F". The penumbra conditionals would directly regulate the boundary. This is why they cannot be part of the rules of a concept that has an unregulated boundary. (Note that this reason is not truth-functional.)

There are two potential problems for that approach. The first asks for the boundaries of the penumbra conditionals, the second concerns a positive reason to regard penumbra conditionals as being rules. The first problem is higher-order vagueness, that the penumbra region has a sharp start- and end point. However, I have already shown in sections (8.3) and (8.4) that higher-order vagueness depends on the forced-march sorites, and that the forced-march sorites is a biased instrument. The crucial step that is missing to defuse the objection is to show that the conditional sorites itself is a form of a forced-march sorites.

What would a forced-march with conditionals look like? I have already stated that the forced-march need not be restricted to any particular statement, such as the paradigmatic "a is F". In principle, it can also be exercised with conditionals of the form "If a_i is F, a_{ii} is F". A forced-march sorites should be comprehensive and detailed, and it should insist on a certain kind of answer or reaction. By considering 10'000 conditionals about the numbers of grains, the conditional sorites is making a comprehensive survey over the whole range of application of heaps in relation to numbers of grains, while holding other factors such as form and material fixed. It is true that one could consider more conditionals, but nothing relevant would be added by considering another 1000 conditionals, since, at that point, it is clear anyway that we should call such a large collection of grains a heap. It is detailed, since it considers all the relevant permutations (numbers of grains) for the issue at hand. And it insists on a certain kind of answer, since the challenge of the conditional sorites is to ascribe a truth-value or semantic status to each of the conditionals. In that way, the proposal to treat the conditionals in the penumbra region not as rules is not committed to sharp boundaries of the penumbra region. We may reasonably refuse to play the game of ascribing to each conditional a determinate semantic status. The conditionals in the penumbra region are not rules, and that region is vaguely delimited.

The second problem is that the criticism "If this is F, that one is F, too!" can also be applied to borderline cases. Therefore, the claim that the conditionals of the penumbra region are not rules seems to be contradicted by what speakers do in unproblematic situations. Indeed, if we follow this line of thought, the paradox reappears, since it

seems that the conditionals of the middle ground are rules just as the other conditionals are rules. My answer to this challenge draws on my account of borderline cases for non-compound statements such as "a is F". There, it is permissible to go either way, and a speaker is entitled to make an *ad hoc ruling*. An *ad hoc ruling* has the status of a rule, but it has that status only temporarily, or in certain contexts. Other speakers of the same language, English, cannot be expected to follow that rule. Equally, my position is that the conditionals in the penumbra region are not rules, and that the other conditionals are rules, and that those conditionals that are not rules can become rules as *ad hoc rulings*. In that way, any conditional can be cited as a rule, but not all of them can be at the same time (by pain of paradox). The idea behind this is that, as far as type-meaning is concerned, the cases in the penumbra region are up for grabs, and that if they come to prominence, speakers can adjust the meaning of the term according to the pragmatic needs of the situation at hand. In that vein, the not-a-rule approach has the tools necessary to deal with penumbral conditionals. If they occur individually or in small groups, we can treat them as part of a borderline context, ready to be dismissed when the context changes. If they occur as part of a group that traverses the boundary, they transcend the guidance that the type meaning provides. Insisting on an answer to the conditionals then invokes the forced-march sorites. But transcending type-meaning in practical contexts can also make the type-indeterminacy apparent and make a sharpening of the concept advisable; be it an *ad hoc* one, or as part of a specialised conceptual system.

8.7. Ben-Yami, or if not Tolerance, what else?

My claim is that Tolerance is not a rule that speakers of standard English follow, and that they are liable to get stuck in the logical mud of the sorites paradox. The question then is: Is there any way to express the vagueness of the boundaries of vague concepts that both captures the irreducible unregulatedness of such concepts and does not get stuck in the logical mud? I will address this question by way of comparing my solution to the one by Ben-Yami. They have a crucial structural analogy. I said that each conditional can be a rule when considered individually (as part of an *ad hoc ruling*), but it is not a case that all of them are rules when considered collectively. For Ben-Yami, the sorites is invalid because "a concatenation of many such [(of the sorites variety)] modus ponens arguments is invalid if it passes the vague boundary between cases where the relevant

concept applies and cases where that concept does not apply" (Ben-Yami 2010, p. 233). The upshot is that each individual *modus ponens* argument is valid, but that a chain of such arguments that passes the vague boundary is invalid. The *modus ponens* arguments are valid individually, and they are invalid collectively.

In that regard, there is a striking similarity between our solutions. At the heart of them lies the acknowledgement that vague concepts need to be understood in a certain way: No individual small change in a dimension δ marks the difference between F and not-F, but a large enough change does. In a way that sounds like a restatement of the paradox. If no small change makes the difference, then a large change cannot mark the difference as well. But this is exactly what we are at pains to elaborate: To understand the boundary in an irreducibly vague way.

Nevertheless, there are three marked differences. First, the not-a-rule approach regards vagueness in the wider framework of rules that speakers follow or do not follow. Second, Ben-Yami is ready to accept all of the conditionals (Ben-Yami 2010, p. 232). At the same time, regarding a "question-and-answer game" where one is asked in a sorites series of each case whether it is a heap, Ben-Yami advices: "There is no use in participating and looking for some sufficient reason to stop at any particular step: there is none. The game as a whole is objectionable and should be avoided." (Ben-Yami 2010, p. 237). In my view, this fails to see what is common in both cases: Our ordinary concepts are not prepared for giving a verdict in a complete sorites series, whether statements of the form $F(a_i)$, $F(a_{ii})$, $F(a_{iii})$ are considered, or whether statements of the form $F(a_i) \rightarrow F(a_{ii})$, $F(a_{ii}) \rightarrow F(a_{iii})$ are considered. In both cases, a forced-march is invoked, which is bound to misrepresent our concepts.

Third, Ben-Yami identifies the problem as one of mistaking an invalid argument for a valid one, whereas I identify it as a problem of accepting an unacceptable premise. This difference amounts to less than one would think. To begin with, validity and entailment are interdefinable: An argument is valid iff its premises entail its conclusion. This means for Ben-Yami that the Tolerance principle together with premise (1) does not entail the problematic conclusion (3). Since (1) is not at issue, this means that we need to understand Tolerance in such a way that it does not entail the problematic conclusion. In that way, the disagreement between Ben-Yami and me boils down to what the standard interpretation of Tolerance, (2), is. I will grant for the sake of argument that one *can* interpret Tolerance in an unproblematic way.

Where we agree is that there are two statements which both resemble Tolerance and

of which one is problematic, and one is unproblematic. He thinks of them as Tolerance, falsely understood, and Tolerance, correctly understood. I think of them as Tolerance, and what we can regard as a rule that speakers follow instead of Tolerance. Regarding the long sorites, I said that each conditional can be accepted as part of an *ad hoc ruling*, but all taken together cannot be accepted. This is why I think that Tolerance is not a rule. Nevertheless, if we consider all cases collectively, what could be the rules that speakers follow? What could an ordinary speaker reasonably reply to the following question: "Can I add a grain and this thing is still not a heap? Can I repeat this?"

26) Oh, you can add a few *buckets* of grains, and do that a few times, and it is still not a heap. But you'll have to stop at some point. – Where do I have to stop? – Just don't overdo it.

In my view, this is the best that an ordinary speaker can reply to such a query. It gives a certain guidance concerning the concept of heap by making clear that a heap can undergo considerable change and still be a heap. Moreover, it considers repeated change with the caveat that too much change will turn a non-heap into a heap. In that way, it upholds the distinction between heaps and non-heaps in the face of possible change. Further, it avoids the universal quantifier in favour of "few". The latter comes of course with an inherent vagueness; we are not told exactly how many times we can throw another bucket into the mix. This is to be expected if vagueness is an irreducible feature of our concepts. Finally, it undermines the question by focusing on buckets instead of individual grains. This reflects that the questions concerning individual grains reads more exactness into the concept than there is. Clearly, this is not a version of Tolerance, but a replacement of it. A proponent of the not-a-rule approach can hold the following view: Tolerance in the formulation of (2) is not a rule, (26) *is* a rule, and every time a speaker accepts an ordinary language version of Tolerance, he is in the grip of the paradox.

Is it problematic to accept Tolerance? We have already seen that close relatives of conditionals such as (23) can be used for criticism. In the same way, a speaker might use a version of Tolerance to criticise somebody who makes a fine-grained distinction between grown-ups and teenagers.

27) You don't grow up in a day.

The problem with (27) is of course that if it is taken in a certain way, it leads into paradox. "OK, you say that this guy who is 16 years and 22 days old is not grown up.

By (27), he will not have grown up if he is 16 years and 23 days, nor 24 days, etc. He will not grow up until he is 80, and further." At this point the speaker may clarify or qualify (27).

28) You don't grow up in a day, it takes years.

The second part of (28) has the function to influence the reading of the first part, not just to contradict its conclusion. It emphasises that there is a distinction, and that the first part should be read in a way that does not undermine the distinction. (27) in the sense of its elucidation (28) can be read as:

29) A distinction between a grown up and a teenager based on a one day difference is always illegitimate. The distinction should be based on years, not days. You should not be counting days.

This amounts to an individualistic reading of (27) in contrast to a collective reading. When considering the one day changes together one after the other, we get into trouble. By only considering one or a couple in a situation, and by upholding the threat of criticism for each such situation, we avoid trouble. Of course, an utterance of (27) can be meant as (29). But to achieve this, the speaker needs to add a qualification with the same effect.

Ben-Yami thinks that the natural or standard reading of (27) is (29), and that the problematic reading is a misinterpretation. However, I think that the unproblematic interpretation requires a bit of work on the part of the speaker; it doesn't come easily. Indeed, if the paradoxical reading becomes difficult to obtain, then it becomes hard to see why we are attracted to the paradox in the first place. But the sorites is such a recalcitrant paradox exactly because we are easily led into paradox, and only with difficulty out of it. Moreover, I think that it is a natural interpretation of (27) that it has consequences for very high numbers of days. My main witness here are related formulations. According to Truism Tolerance, if a man with a height of n centimetres is tall, a man with a height of $n+1$ centimetres is tall as well. This statement has consequences for very high numbers of centimetres, to the extent that one might wonder whether it does not overreach. Surely, a three-metre man is tall. But a three-kilometre man? Is this a man? And if my friend tells me "I will do it tomorrow", and he tells me every day the same, I start to suspect he will never do it. That is why speakers cannot

just accept versions of Tolerance such as (27); they need to do some work to defuse the luring paradox, for instance with (28). They can accept a qualified version of Tolerance, but not Tolerance itself. Tolerance is not a rule they follow.

8.8. The attraction of the sorites

The sorites paradox is a fallacy in the sense to be found in Merriam-Webster: It is a plausible argument with false premises or invalid inference. According to the not-a-rule approach, the fault is to be found with the premises. For the short version, Tolerance is a premise that looks appealing but is false. For the long version, the long list of conditionals looks appealing but there is no situation in which *all* of the conditionals are rules without that the vague concept would acquire a related but new meaning.

For each version there are premises that are appealing but false. Why are they appealing? And why is it so difficult to see past their trickery? I see four interconnected considerations. The contrast between the two versions is often exaggerated. Of course it is true that the Tolerance principle entails the ten thousand conditionals, but not vice versa. The most one can get from the reversed direction is a restricted Tolerance, which does not hold for, say, all natural numbers, but only for the natural numbers k to m. Nevertheless, there are strong commonalities. For one thing, the ten thousand conditionals and Tolerance are justified in a similar way. Typically, we do not judge the truth of each conditional individually, but we consider a few samples and then generalise to the whole ten thousand. (To my knowledge, no one has ever *written down* the whole set without the help of a notational device that says "and so forth".) In the same way, Tolerance can be justified by appealing to individual conditionals that are entailed by it. After we see that there is no reason to oppose the generalisation, we accept it. For another, both Tolerance and the ten thousand conditionals cover the whole range of a sorites series and impose themselves by their apparent unavoidability.

The first reason why the sorites is so appealing is that expressions that are closely connected both to Tolerance and to the set of ten thousand conditionals are used to criticise and justify uses of words. Consider the following situation. Two children, Adam and Benjamin, want to go on an roller-coaster. Adam is 150 cm, and Benjamin is 1 centimetre smaller. The roller-coaster has a sign which says: "Small children are not allowed". Their father tells them that Adam is allowed to go in, but not Benjamin. Benjamin complains that he is only a little bit smaller than Adam, and if Adam is allowed

to go, he should be too. Benjamin clearly has a point, the difference in height between them is not sufficient to warrant the application of "small child" to one but not the other. Nevertheless, this need not be the end of the story. The father then may point out the fine print, in which a small child is defined as 149 cm or less. In that way, the operator of the roller-coaster, or the legislator, has made an *ad hoc ruling* as to whom is to count as a small child. And while this is arbitrary, and *pro tanto* unjustified, an *ad hoc ruling* is justified as an *ad hoc ruling*. In such a case, Benjamin's criticism is of no avail.

This shows that Tolerance and the conditionals are not a total fabrication. They have a firm basis in the normative practice of speakers, even if the favourite expression is not in terms of numbers of grains or centimetres, but uses indexical terms. At this point, a second seduction takes place: generalisation. Individual conditionals can be used in justification and criticism. Even worse, *any* individual conditional can be used for such a purpose. Worse again, there is no obvious reason why any of the conditionals should not be true, or why we should think that any of the conditionals is not a rule that is in force for our concept. Given all these reasons, it is absolutely natural and persuasive to generalise from a few individual conditionals to a whole set of conditionals or Tolerance: All of these conditionals are true and rules that are in force for our vague concept.

But of course if all conditionals are rules, they lead to the unsavoury conclusion that even a two-metre child is a small child. At this point, it becomes difficult to find a way out. We have reason to accept the conditionals, but they lead to paradox. What is operating in the background is a poor and unreflected understanding of the status of these conditionals. This is the third consideration. It is already a rarely taken and important step to recognise the conditionals as purported rules, and to frame the discussion as one of revolving around the questions: In what way do the conditionals or Tolerance surface in our explanations of the meaning of expressions, in our justifications and criticisms of applications and uses of these expressions, and how do we apply and use these expression? But a further step that needs to be taken is to realise an important feature of rules: They can leave things unregulated; there might be no straightforward application in certain situations. Regarding vague concepts, the most important thing that is not regulated is the boundary itself. There are no rules that tell us how tall a small child can maximally be in centimetres, how many grains need to be assembled to make a heap, or how many pranks one needs to pull off per month in order to count as a rascal. Connected to this is the idea that a sentence P might not be a rule that is in force without not-P being a rule. In other words, the status of a (potential) rule of P allows

that it can be negated externally. This has its most straightforward application in the case of Tolerance. Tolerance can be false – because of its status as a potential rule – without that the internal negation of Tolerance is a rule, that is, without that we are thereby committed to sharp boundaries. But it also plays a role in the case of the ten thousand conditionals. Without the option of an external negation, a conditional can only be false if its antecedent is true and its consequent false, again implying a sharp boundary. Nevertheless, in the case of the conditionals, a harder problem lurks. We are able to use *any* as a criticism, which seems to suggest that *any* conditional should be a rule.

The way out of this problem takes recourse again to the idea that not everything is regulated, and that our common vague concepts are not prepared to deal with *any* situation. A first situation where the rules give out are borderline cases. The reasons pull with roughly equal force in both directions and do not determine a correct verdict. A second such situation arises if we are asked to consider all cases along a sorites series. This set-up is bound to lead to sharp boundaries whether it is about statements of the form "$F(a_i)$" or "$F(a_i) \rightarrow F(a_{ii})$". But such situations are highly constructed. In the normal course of events, sorites series are never complete: They trail off at some point, leave gaps, cluster in groups, etc. It is little wonder then that our vague concepts are not prepared to deal with such artificial situations, and that the picture that the rules in force draw is in conflict with such a situation. The conflict arises because the rules we know do not draw a boundary, and if there were one it would be something we should be informed about. However, the artificial situation demands that such a boundary be drawn. That is why we cannot engage with a complete sorites series without distorting, warping, and altering our concepts. So we should object to the set-up of the questioning, both in the case of simple verdicts "This child is small", and in the case of conditionals. (And I have shown in section (8.6) what sorts of rules we can and do follow for vague concepts for all cases.)

Someone might not be satisfied by such an answer. It might strike her as evasive. At that point, she is likely to confront us with a forced-march sorites, and thereby force an answer to the semantic status of each statement in a sorites series. This brings me to the fourth consideration: What underlies this discussion is a deep-rooted logical prejudice. The demand for clarity is an important aspect of using reason. Logic is seen as the prime exemplar of clarity. But the way clarity is applied to the logic of vague concepts incurs two errors. First, it confuses a demand concerning speech with a demand concerning

type-meaning. In a criminal investigation, it is the job of the interrogator to get clear answers to his questions, to prevent evasion. "Did you tie him up or not?" – "Kind of." – "What did you do exactly? Did you handcuff him? Did you use a handcuff to tie him to a radiator? Did you use rope?" In that way, clarity attaches to speech acts, not to type-meaning. Second, the demand for clarity concerning speech acts is highly context-dependent. Tailors use a more detailed colour scheme than the one provided by non-jargon English, and borderline cases are easily amenable to *ad hoc rulings*. To straighten out the irreducible indeterminacy of the type-meaning of vague concepts distort those concepts if the goal is to describe them. And without a pragmatic purpose, the logical demand adds complexity without solving any problem.

Conclusion

How does the not-a-rule approach fare if put under strain by scrutinising borderline cases and statements about them? The argument of this essay shows that the approach becomes more complicated, but that it can deal with any new problem. Borderline cases are interpreted as an interplay between an incompletely regulated type-meaning and the requirement to avoid unclarity in one's speech. For borderline cases of F, the fully contextualised type-meaning of F does not determine clearly whether it is F or not-F. This is not a full-fledged category, but a direct indeterminacy in the type-meaning that stems from incomplete regulation. In such cases, a speaker can with equal justification apply F or not-F, though not in the same situation without contradicting himself. An application in borderline case may be regarded as an *ad hoc ruling*. As such, it does not sport unclarity, does have a truth-value, and the speech act is completely unproblematic. In other cases, not enough work is done for an *ad hoc ruling*, and the hearer may reject the speech act as infelicitous. Importantly, speakers may as well problematise the lack of decisive reasons for application and ascend to a meta-level. The speech act interpretation of statements about borderline cases applies as well to penumbral connections. Negated instances of the Law of Excluded Middle, of Bivalence, and of the Law of Non-contradiction can be used for problematising borderline cases, asserted instances for epistemic encouragement. Moreover, Truism Tolerance can be regarded as a rule.

The conditional sorites constitutes a difficult quandary. Each individual conditional may potentially serve as a justification or criticism of an application. But all condi-

tionals taken together lead to paradox. The way out, as I see it, relies on seeing the boundary as unregulated. Accordingly, conditionals in the penumbra region are not rules. Nevertheless, they may be used in *ad hoc rulings*, which creates the impression that they should be treated in the same way as the other conditionals. Moreover, the unregulated boundary creates an indeterminacy that stubbornly resists to be straightened out in terms of a comprehensive and detailed perspective. The attempt to insist on a straightening out through the forced-march sorites is beset by two deep problems. First, the irreducible indeterminacy manifests itself in the troubles of higher-order vagueness, where a straightened out indeterminacy leads to a multiplication of sharp boundaries, and ultimately, overprecision in the form of a great number of categories. Furthermore, to insist on an answer to each individual question in the paradoxical context of the sorites on a logical prejudice.

Endnote

Indeterminacy is a ubiquitous and silent feature of concepts of a natural language such as English; if the facts (described in one set of terms) lie in a particular way, the meaning and rules of a concept do not determine a correct application of that concept; the rules do not cover these cases. One-dimensional vagueness is a case of indeterminacy that is easy to construct: Slide along a similarity dimension towards the boundary between F and not-F, and you will discover borderline cases. The construction of cases of multi-dimensional vagueness requires more work, but they are equally widespread. Is Brazilian football a borderline case of a religion? There is belief in figures with supernatural powers worthy of worship, and ritualistic behaviour (cf. Bueno/Colyvan 2012, p. 30). Abnormal cases stretch the rules further, where the world behaves in unprecedented ways, which reveals other limitations of a language geared towards the normal. But of course the absence of rules for abnormal and infrequent cases impedes in no way the usefulness of language.

In an immanent critique of the use of words of a speaker, such gaps in the rules are not *per se* worthy of criticism. Such an immanent critique may require an investigation into the rules that the speaker follows; if in some cases no rules are forthcoming, this is the result. But the ultimate goal of philosophy as immanent critique is not to gain an overview over the rules of a language, but to solve philosophical problems that speakers of that language have. For that purpose, an investigation of use can reveal gaps or inconsistencies in the explanations, and thereby show that what the speaker is inclined to say lacks a clear sense, and for that reason, one might say, is nonsense. But there is another case: What the speaker takes to be a rule that he follows turns out to be no rule when compared to his employment, and normative interventions regarding that term (explanations, justifications, criticism). In that case, the speaker is mistaken, but does not speak nonsense. Therefore, the immanent critique has at least two modes: the

demonstration of nonsense; and of a false description of use. Moreover, the investigation of use is, at least potentially, specific to individuals. It does not matter what the majority of a linguistic community does, or what authoritative institutions prescribe. Only if there is a problem within the use, in the extreme, of the person under investigation, does a philosophical problem arise. But then, there is no easy standard with which his use can be compared and declared nonsense. Only a painstaking investigation into his use in the form of an immanent critique touches upon a philosophical problem. This then is the core of a Wittgensteinian method. It is not a simple recipe to cook up things and separate sense from nonsense, but calls for scrupulous and creative attention to the sources, motivations, arguments, and the use of the person that has a philosophical problem.

The sorites is a case where no nonsense is involved; but the landscape of the language we speak is set to usher the speaker along a false path; she is prone to misrepresent the rules that she follows. For one thing, that declarative speech acts can have different statuses is a familiar phenomenon, even if it is completely unfamiliar as a part of an explicit description of use. The statuses under consideration are that of being an empirical statement, and being an expression of a rule. Consider a game of chess where the players introduced an *ad hoc* variant rule. We know quite well how to find out what the rule is, and how to distinguish it from advice how to win the game, or descriptions of the state of the board "The white king is at E1". Without that distinction in mind, there seem to be only two ways to react to Tolerance: Accept it and be mired in paradox, or deny it and be committed to sharp boundaries. But reflecting on the status of Tolerance as a potential rule reveals a third way: Tolerance is not a correct description of our use, and is false because it purports to be a rule that we follow, but is not. This amounts to an external negation of the purported rule. And it does not follow from "It is not a rule that p" that "It is a a rule that not-p". To say that Tolerance is not a rule does not imply sharp boundaries.

For another, the acceptance of Tolerance as a rule has a clear, if insufficient basis in use: There is the closely related Truism Tolerance (see p. 185), and more importantly, conditionals associated with Tolerance such as "If Peter is old enough to go on the roller coaster, I am too!" that *can* be used in the normative role of criticism. The conditional sorites picks up this thought. If a single conditional is recognised as a rule, and the other conditionals could be employed similarly, then a series of ten thousand conditionals should be recognised as rules, too. These ten thousand conditionals then lead into paradox.

The way out is based on two ideas. The first is that in a borderline context, an *ad hoc* decision can be taken for an individual conditional: To regard it as an *ad-hoc* rule. That does not change the fact that the conditionals in the middle ground are up for grabs, precisely because of the inherent indeterminacy of our concepts. The second idea is that to insist on an answer for each conditional of a sorites series is nothing else than a case of a forced-march sorites. It leads to a comprehensive and detailed perspective by forcing the speaker to provide an answer in each case. But such a perspective necessarily distorts an indeterminate concept and embodies a unjustified prejudice against unregulatedness in language.

This, I think, is the hardest form of the paradox. What lies at the heart of it is a generalisation that takes us from an acceptable description of our use (any conditional can be accepted individually), to an unacceptable one (all conditionals can be accepted together). There is a conflict between a comprehensive and detailed perspective and the indeterminate reality of our language. The use of a word such as "heap" has developed to fulfil various communicative purposes: We give orders to pour and remove heaps of gravel, count heaps of spice, and admonish children who throw their cloths into heaps. In all those cases, the boundary never comes into view. It is of little wonder that our natural language does not have, and does not need to have, rules that would decide boundary cases of affiliation. The comprehensive and detailed perspective urges us to ask questions about our concepts whose answer would require the concepts to be tidied up and rectified, thereby changing them. If we leave our concepts as they are, their answers will appear to be incomplete from a comprehensive and detailed perspective. By stepping down into the thick of life, however, we can appreciate that the concepts answer the questions that are at home in that alternative perspective. And if they do not, we can amend and change them for pragmatic purposes.

Historically, interest in vagueness thrived when interest in logic thrived (cf. Williamson 1994, p. 36). In the nineteenth century, Frege revived the interest in the sorites paradox, and spoke of concepts with unsharp boundaries. These concepts, however, encompass all kinds of indeterminacy, any threat to the *Law of Excluded Middle*. In contrast, in the contemporary debate, a concept with unsharp boundaries involves a transition zone. It is important to keep in mind that difference in meaning when evaluating early Analytic Philosophy, in particular Frege and Wittgenstein, and probably Russell (1923) as well.

The logical atomism of the *Tractatus* dreamed of a language that would be determinate in *any* state of the world. For that purpose, an analysis of any humdrum statement

would ultimately involve simple objects, the alphabet of what is possible. These objects would exist necessarily, and make up the metaphysical underpinning of language. An underpinning, *nota bene*, about which one could not speak according to the *Tractarian* theory of language; a move that notoriously condemned the whole book itself to the fate of gibberish. With that treatment of indeterminacy in view, the Wittgenstein of the *Investigations* brought forth an argument that denied the possibility of such a completely determinate language. Doubts and misunderstandings can arise regarding any rule, including the rules that regulate the application of another rule. But then, any set of rules that is thought to be completely determinate can be thrown off balance by a further doubt. If there were simple objects, there may be doubts on how to apply signs that refer to them. But then, the idea of a completely determinate language turns out to be a chimera.

Moreover, the *Tractarian* version of analysis reaches into deep levels that play no role in the explanations, justifications, and criticisms of speakers. A description in terms of molecules is no help in deciding whether a broom is in the corner or not. And since the rules that a speaker follows are partly constituted by such normative interventions, there is little reason to think that the rules that he follows involve distributions of molecules. Rather, whether a broom is in the corner depends on whether the object in the corner can fulfil its function of sweeping the floor, whether it looks like a broom, whether it is made of the right materials, etc. The idea of a *Tractarian* deep-level analysis therefore also rests on a misguided conception of what it is to speak a language. And if the actual and potential employment and normative interventions of a speaker are indeed constitutive for the rules he follows, for (the variant of) the language that he speaks, it becomes a real possibility that his use may leave blank spaces. That his, and our, use does indeed leave such blank spaces can be revealed by an investigation into the use of words in the light of recalcitrant behaviour of the world, by sliding along a similarity dimension, or combining a variety of criteria freely.

This leaves us with a double result. On the one hand, philosophical questions for the nature and demarcation of life vs. mere matter, life vs. death, rational animals vs. non-rational animals, embryo vs. full-fledged human being, etc. may lead us to discover blank spaces on the map of the language that we speak. Pressing further then cannot fill those spaces without changing the conceptual basis of our investigation (even if, as in the case of "rational animal", the debate is framed in technical terms – for these terms have connections to non-technical vocabulary). On the other hand, merely acknowledging

the blank spaces may not be enough in cases where we need to decide what to do – for instance, how to treat brain-dead persons, or if and until when to allow abortion. In such cases, there is a pragmatic pressure to fill in the blanks and thereby change our concepts.

It would be a noteworthy task to engage with such pragmatic questions as they come up in our society, and try to find philosophically justified and pragmatically acceptable answers. By engaging with a series of such questions instead of a single one, one could compare and contrast the procedure and the constraints that lead to successful answers. In that way, the more abstract question in how to proceed with pragmatic questions in blank territory could be answered in light of engaging with more concrete problems. Similarly, the way forward for better understanding a successful Wittgensteinian method in philosophy would be to solve particular philosophical problems, and to reflect on one's own procedures in doing so. In that way, one could get a better grip on what role is played by the invention of *problem-oriented* methods, in what ways the sources of confusion influence a philosophical investigation, and which other factors are relevant in the resolution of the problems. Thereby, in a collaborative effort, an arsenal of methods and strategies for the resolution of philosophical problems could be assembled, where these methods and strategies would be partially competitive, and partially complementary.

> How much pectin does one have to add
> to a pot of banana jam
> to turn it into banana jelly?
> Two tea spoons should do it.
>
> But how much exactly?
> Which crystal of pectin does it?
> The crystal that is not a crystal.

Appendix: Varieties of Indeterminacy

Over the course of the book, several kinds of indeterminacy and related phenomena have been discussed. This appendix brings them together and comments concisely on them. The most general kind of indeterminacy is the Fregean one.

Fregean Indeterminacy: A concept F is unsharp iff, for at least one object a, it is neither determinate that a is F nor determinate that a is not F.

This includes incomplete definition, one-dimensional vagueness, and multi-dimensional vagueness.

Incomplete Definition: A concept F is incompletely defined iff, for some object a, it is neither defined that a is F nor defined that a is not F.

Example:

a) A number n is nice if $n < 13$
b) A number n is not nice if $n > 17$

The definition of "incomplete definition" (of concepts) and of "Fregean Indeterminacy" differs only in that "determine" is replaced by "define". Fregean Indeterminacy (and its generalisations) is the only kind of indeterminacy that includes incompletely defined concepts, such as "nice number", where nothing is defined for the numbers between 13 and 17.

The concept-pair of one-dimensional and multi-dimensional vagueness (or degree and combinatory vagueness) does not only apply to concepts, but also to other kinds of words such as proper names. Moreover, because the modern discussion of vagueness (in the 20th century) is centred on degree vagueness, I sometimes call it "modern vagueness".

One-dimensional Vagueness: Cases of indeterminacy can be constructed by proceeding along one similarity dimension while holding other similarity dimensions constant.

Multi-dimensional Vagueness: Cases of indeterminacy can be constructed by combining several dimensions of similarity.

One and the same word can exhibit both one- and multi-dimensional vagueness, leading to four combinations: no vagueness (acute angle), both (democracy), only one-dimensional vagueness (heap), and only multi-dimensional vagueness (game).

A source of vagueness that is perpendicular to the one- and multi-dimensional distinction is the distinction between abnormal and normal cases. A collection of 43 grains is an ordinary and normal phenomenon; nevertheless, it is a borderline case of "heap". Equally, say, Brazilian football is an actual and ordinary phenomenon, but it may be regarded as a borderline case of "religion" due to multi-dimensional vagueness. It is logically possible that there is such a thing as half-frozen water with a consistency between ice and liquid water; but as a matter of physical fact it does not exist. This would be a case of abnormal, one-dimensional vagueness. And Wittgenstein's spooked chair is an example of abnormal, multi-dimensional vagueness.

Abnormal Vagueness: Cases of indeterminacy can be constructed by considering cases that are unlikely to occur in the normal course of events.

Fregean Determinacy is exclusively formulated for concepts. A more general version is formulated for statements, and thereby does not make any claims about the structure of statements (whereas Fregean Determinacy implies that one of the logical forms in statements are concepts). In particular, the *Tractatus* does not recognise the Fregean distinction between concepts and objects as one that concerns logic, which is about the general conditions for a statement to have sense sense (be truth-apt). (Though Fregean concepts and objects may be discovered in the application of logic, which reveals the structure of statements). Propositional Determinacy in effect amounts to Bivalence, because reference to every possible state of the world is tacitly included in the latter.

Propositional Determinacy: A proposition is true or false (has a truth-value) in any possible state of the world.

Bivalence: A proposition is either true or false.

However, the equivalence between Propositional Determinacy and Bivalence requires that they should be understood to hold for the same level: that of type-meaning, or that of speech. In my view, they hold for the level of speech, and that it does not make sense to speak of truth-values of type-meaning sentences. However, before the advent of Speech Act Theory and the category of infelicitous statements, it was tempting to conflate these two levels. Frege and the *Tractatus* seem to have understood them as holding for the type-level. If so, their principle of Bivalence differs from the one that I find acceptable. One way to distinguish between these versions would be to formulate them for propositions, and for statements, respectively.

Propositional Determinacy together with the picture theory which includes the referential doctrine of the meaning of names, imply that the structure of a statement must terminate in names for simple (*Tractarian*) objects.

Unspecificity: A statement is unspecific iff it is true in more than one situation.

According to the *Tractatus*, even a statement that appears to be very specific is actually unspecific, for instance:

1) On the 15th of August 1945 at 9:01 pm, the Japanese Emperor Hirohito brushed his lower left third molar with his favourite tooth brush.

In the *Investigations*, the ideas of Propositional Determinacy and Fregean Indeterminacy are expressed in terms of rules.

Rule Determinacy: The application of a word is regulated on all sides.

Of course, the *Investigations* argues that this is not even possible. Moreover, it relates the resulting idea of unregulatedness to the dynamic aspect of language. If the primary objective of the rules for ordinary concepts is to regulate frequent and normal cases, this suggests a dynamic conception of meaning.

Dynamic Meaning: Not every change in the constitutive rules for the expression E to E^* justifies the verdict that E and E^* are not the same expression.

According to this conception, speakers can adapt to occurring borderline cases by altering rules and introducing new rules, thereby not necessarily changing their concepts and expressions.

In the period between the *Tractatus* and the *Investigations*, Wittgenstein regarded blurredness (*Verschwommenheit*) as an inherent feature of experience, understood in a phenomenal way.

Blurredness of Experience: A phenomenal concept or experience is blurred iff it corresponds to an inexactly bounded class of physical "correspondings".

For instance, a seen circle corresponds to a class of physical shapes that resemble a geometrical circle. But the class itself is inexactly bounded. With the advent of so-called the private language argument, the construction of blurred boundaries as a feature of phenomenal or visual space receded.

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